Unit Plan

Secrets of the Pythagoreans

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For
University of Rhode Island
EDC430
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Unit Plan Summary

This unit is developed for ninth grade students in a math foundations class. The student skills are below grade level with the focus being to strengthen the base skills for entry into mandatory algebra. The unit is designed to meet the standards for GSE/GLEs 6-8 and grade 10. As a result the unit is designed to bring the skills from an eighth grade level or lower to a ninth grade level. The unit utilizes the structure of right triangles and their special properties to connect many geometric concepts and ideas that flow naturally from the exploration of the Pythagorean Theorem. This unit addresses a number of concepts in geometry, through an inquiry based learning approach. The students seem to be at a Van Heile level two, for which the Pythagorean Theorem is appropriate. The goal is to encourage the students to discover the properties and formulas, making the connections for future deductive reasoning, rather than memorizing formulas and properties.

The theme is designed to demonstrate how the ancient Greek, Pythagoras contributed to the math we use today. In an attempt to relate to the students, the unit investigations are designed as challenges. If the students meet the challenges presented to them by demonstrating understanding, they will be inducted into the secret society of the "Pythagoreans". The inductees of the secret society (all students) will have a small celebration at the end of the unit post summative assessment. The historical aspect of the unit demonstrates some integration of other disciplines. It also creates some continuity serving as a story link through the lessons.

The flow of the lessons begins with the introduction of the right triangles in the orthogonal system of the coordinate grid, or Cartesian plane. Students begin by measuring the hypotenuse or distance between two points. The second lesson on squares and square roots is a pre-requisite skill for the unit. It will be determined in lesson 1 whether this lesson needs to be taught in depth or more as a review. Students then move to discovering the special property of right triangles, namely the Pythagorean Theorem. Lesson four extends the Pythagorean Theorem exploration to examining the relationship for obtuse and acute triangles. Pythagorean triples are emphasized as these are useful for future geometric reasoning. In lesson five, the students move back to the coordinate plane to apply the Pythagorean Theorem and derive the distance formula. Lesson six addresses right triangle similarity and proportional reasoning. Lesson seven applies the concepts of similar right triangles and introduces the concept of indirect measurement. The hope is that this lesson can be conducted outside for movement and fresh air. A historical connection is made to Thales, a discoverer of indirect measurement and similar triangles. Lesson eight will extend for two days covering various word problems and geometric deductive reasoning problems with application of the concepts learned. The last lesson touches upon the ultimate Pythagorean secret of irrational numbers. Since the students will be heavily into geometry, this unit will reinforce or teach with depth anything the students have learned by the time this unit can be administered.

The lesson on irrational numbers is included in the unit as an extension and can be used in the future. Another natural extension, right triangle trigonometry, could be addressed within this unit as well.

A chart of the high level RI Beginning Teacher standards is provided. Those standards met within this unit plan are checked. Whether a new teacher or not, these standards should apply to all teachers, and I will strive to do my best to incorporate those ideals embodied in the standards in my practice to the best of my ability.

Diversity was addressed through different learning styles in the multiple intelligences philosophy. Activities and explorations were designed to engage students and provide different ways for students to learn. The use of differentiation acknowledges the diversity of the students.

Technology use was mainly in the form of online applets that demonstrate and summarize concepts. Online video clips and calculators round out the approach.

Heavy use of student centered discussions will help reveal student needs, misconceptions or strengths, allowing an efficient lesson to be delivered.

Unit Plan Goals

- Develop strategies for finding the distance between two points on a coordinate grid
- Understand and apply the Pythagorean Theorem
- Understand and apply the triangle inequality theorem
- Understand and apply the distance formula
- Understand and apply the concept of similar right triangles
- Extend understanding of the number system to include irrational numbers

Guiding Questions

How did ancient cultures contribute to mathematics?
How is geometry part of the world?
How do we solve geometric problems?

Unit Plan Overview

#	Lesson Title	Lesson Duration	Lesson Objectives
1	*Pythagorean Challenge -1 Pythagoras, Taxicab Driver*	1 day	• Activate prior knowledge about the area of a square, calculating square roots, coordinate plane and coordinate plane geometry • Create a table to organize observations • Draw conclusions from the data collected • Justify the observations • Explain/communicate the process used during the exploration process • Discover the relationship of the sides of a triangle and the triangle inequality theorem
2	*Pythagorean Challenge -2 The secrets of squares and the square roots*	½ day	• Understand the relationship between squares and square roots
3	*Pythagorean Challenge 3 - Pythagoras and the magical property of right triangles*	1 to 1½ days	• Discover the relationship between the areas of squares that form a right triangle • Develop an introductory understanding of geometric proof • Activate prior knowledge about the area of a square, triangle classifications • Classify, illustrate and explain triangle types with focus on right triangles • Apply the Pythagorean Theorem in determining the hypotenuse (the lesson will be extended to find the missing side and applications) • Create a table to organize observations • Draw conclusions from the data collected • Justify the observations • Explain/communicate the process used during the exploration process
	Quiz_1	1/3 day	• Assess use of Pythagorean theorem in finding a missing side of a triangle
4	*Pythagorean Challenge 4 - Acute and Obtuse Triangles Pythagorean Triples*	1 day	• Classify triangles (acute, obtuse and right) through the $a^2 + b^2 \ (>, < \text{ or } =) \ c^2$ relation. • Understand and utilize Pythagorean triples • create a table to organize observations • draw conclusions from the data collected • justify the observations • explain/communicate the process used during the exploration process

5	*Pythagorean Challenge 5 – Going the distance*	1 day	• apply the Pythagorean Theorem • derive the distance formula • calculate the shortest distance between two points on the coordinate grid
	Quiz_2	1/3 day	• Assess understanding of the distance formula
6	*Pythagorean Challenge 6 - Special Right Triangles Similarity, Proportions and Scale*	1 day	• Understand special right triangles 30-60-90 and 45-45-90 • Develop an understanding of similarity • Review congruence • Apply similarity to find lengths in a similar triangle • Apply proportion to solve problems involving scaled right triangles • Explain/communicate the process used during the exploration process
	Quiz_3	1/3 day	• Assess understanding of similarity
7	*Pythagorean Challenge 7 - Measuring the height of a Grecian temple*	1 day	• Application of similar triangle proportions for indirect measurement • Explain/communicate the process used during the exploration process
8	*Pythagorean Challenge 8 - Using the Pythagorean Secrets*	2 days	• Analyze triangles and other shapes with right triangles • Apply the Pythagorean Theorem to find missing sides to determine perimeter or area of two dimensional shapes • Explain/communicate the process used during the exploration process • Create a study guide for use on the exercises and the summative assessment
9	*Pythagorean Challenge 9 – The Ultimate Pythagorean Secret*	1 day	• Identify irrational numbers • Understand relationship to rational numbers and the real number system • Locate irrational numbers on a number line
10	*Pythagorean Monument Performance Assessment*	1 day	• Summative Assessment
	Pythagorean Hood Induction Party	½ day	• Small celebration, toga party, games to further emphasize concepts learned.
	TOTAL	12 days	

RI Mathematics Standards

State of RI Mathematics Standards	Lessons
M(N&O)–8–2 Demonstrates understanding of the relative magnitude of numbers by ordering or comparing rational numbers, common irrational numbers (e.g., ,), numbers with whole number or fractional bases and whole number exponents, square roots, absolute values, integers, or numbers represented in scientific notation using number lines or equality and inequality symbols. (Local)	9
M(G&M)–8–2 Applies the Pythagorean Theorem to find a missing side of a right triangle, or in problem solving situations. (Local)	3, 5
M(N&O)–8–4 Accurately solves problems involving proportional reasoning (percent increase or decrease, interest rates, markups, or rates); multiplication or division of integers; and squares, cubes, and taking square or cube roots. (Local)	1,2,4,7,5
M(N&O)–8–6 Uses a variety of mental computation strategies to solve problems (e.g., using compatible numbers, applying properties of operations, using mental imagery, using patterns) and to determine the reasonableness of answers; and mentally calculates benchmark perfect squares and related square roots (e.g., 12, 22, … , 122, 152, 202, 252, 1002, 10002); determines the part of a number using benchmark percents and related fractions (1%, 10%, 25%, , 50%, , 75%, and 100%) (e.g., 25% of 16; of 330). (Local)	3, 9
M(G&M)–8–5 Applies concepts of similarity to determine the impact of scaling on the volume or surface area of three-dimensional figures when linear dimensions are multiplied by a constant factor; to determine the length of sides of similar triangles, or to solve problems involving growth and rate. (Local)	6, 8
M(N&O)–10–2 Demonstrates understanding of the relative magnitude of real numbers by solving problems involving ordering or comparing rational numbers, common irrational numbers (e.g., ,), rational bases with integer exponents, square roots, absolute values, integers, or numbers represented in scientific notation using number lines or equality and inequality symbols. (State)	1
M(G&M)–10–2 Makes and defends conjectures, constructs geometric arguments, uses geometric properties, or uses theorems to solve problems involving angles, lines, polygons, circles, or right triangle ratios (sine, cosine, tangent) within mathematics or across disciplines or contexts (e.g., Pythagorean Theorem, Triangle Inequality Theorem). (State)	3, 4,5
(N&O)–10–4 Accurately solves problems that involve but are not limited to proportional relationships, percents, ratios, and rates. (The problems might be drawn from contexts outside of and within mathematics including those that cut across content strands or disciplines.) (State)	6, 7, 8
M(G&M)–10–5 Applies concepts of similarity by solving problems within mathematics or across disciplines or contexts. (State)	6, 7
M(G&M)–10–6 Solves problems involving perimeter, circumference, or area of two-dimensional figures (including composite figures) or surface area or volume of three-dimensional figures (including composite figures) within mathematics or across disciplines or contexts. (State)	8

M(G&M)–10–9 Solves problems on and off the coordinate plane involving distance, midpoint, perpendicular and parallel lines, or slope. (State)	5

RI Professional Teacher Standards

The following table indicates the RI Professional Teacher Standards exhibited in this unit plan.

Std	RI Professional Teacher Standard Description	Demonstrated in Unit Plan
1.0	Teachers create learning experiences using abroad base of general knowledge that reflects an understanding of the nature of the world in which we live.	√
2.0	Teachers have a deep content knowledge base sufficient to create learning experiences that reflect an understanding of central concepts, vocabulary, structures, and tools of inquiry of the disciplines/content areas they teach.	√
3.0	Teachers create instructional opportunities that reflect an understanding of how children learn and develop.	√
4.0	Teachers create instructional opportunities that reflect a respect for the diversity of learners and an understanding of how students differ in their approaches to learning.	√
5.0	Teachers create instructional opportunities to encourage students' development of critical thinking, problem solving, performance skills and literacy across content areas.	√
6.0	Teachers create a supportive learning environment that encourages appropriate standards of behavior, positive social interaction, active engagement in learning, and self-motivation.	√ 6.1, 6.2, 6.3, 6.4, 6.6
7.0	Teachers work collaboratively with all school personnel, families and the broader community to create a professional learning community and environment that supports the improvement of teaching, learning and student achievement.	
8.0	Teachers use effective communication as the vehicle through which students explore, conjecture, discuss, and investigate new ideas.	√
9.0	Teachers use appropriate formal and informal assessment strategies with individuals and groups of students to determine the impact of instructional on learning, to provide feedback, and to plan future instruction.	√
10.0	Teachers reflect on their practice and assume responsibility for their own professional development by actively seeking and participating in opportunities to learn and grow as professionals.	
11.0	Teachers maintain professional standards guided by legal and ethical principles.	

Accommodations and Modifications

- One student needs to be seated close to the front for any board work.
- Many of the students have IEPs but no special modifications have been indicated other than extra time on assessments. Many of the issues will manifest in behavioral ways.
- There is an attempt to refer back to prior knowledge for consistent review and connections in hopes of providing an opportunity for depth of learning. This is helpful to all students but is it also an accommodation for AD/HD students.
- Connecting the usefulness of the tools the students are learning and how they may be useful in a job in the future.
- Writing the summary information in the notebooks, helps scaffold the students on how to take notes. The information is there for reference for any students with AD/HD as they may have to refer back to their notes.
- Study guides are helpful for revisiting the important information and some students may require the use of the study guide on an assessment.

Diverse Learner Plan

- There are hands-on activities which are good for those kinetically inclined students.
- There is a visual element with the online video clips and applets.
- Flexible groupings will be used during this lesson and can be arranged by the instructor as needed. The instructor can use flexible grouping strategies to accomplish the task at hand. This will meet social needs as well as provide an opportunity for peer tutoring.
- Advanced students will be engaged with deeper questioning to allow them to attempt to use a higher order thinking skill of analyzing and solving the problem.
- Homework allows the student time to work at his/her own pace and increase understanding.
- Other specific diversity issues are addressed within the lesson plans.

Lesson Plan Set

Lesson 1 – Pythagoras, Taxicab Driver

Grade/Content Area	For ninth grade math students in a pre-algebra (math foundations) class.
Lesson Title	***Pythagoras, Taxicab Driver***
State Standards: GLEs/GSEs **National Content Standards**:	RI Standards: M(G&M)–10–2 **Makes and defends conjectures, constructs geometric arguments, uses geometric properties, or uses theorems to solve problems** involving angles, lines, polygons, circles, or right triangle ratios (sine, cosine, tangent) within mathematics or across disciplines or contexts (e.g., **Pythagorean Theorem, Triangle Inequality Theorem**). (State) M(N&O)–8–4 Accurately solves problems involving proportional reasoning (percent increase or decrease, interest rates, markups, or rates); multiplication or division of integers; and squares, cubes, and taking square or cube roots. (Local) M(CCR)–8–1 **Students will communicate their understanding of mathematics** and be able to: • Articulate ideas clearly and logically in both written and oral form. • Present, share, explain, and justify thinking with others and build upon the ideas of others to solve problems. • Use mathematical symbols and notation. • Formulate questions, conjectures, definitions, and generalizations about data, information, and problem situations. NCTM Communication Standard • communicate their mathematical thinking coherently and clearly to peers, teachers, and others; • analyze and evaluate the mathematical thinking and strategies of others;
Context of the Lesson *Where does this lesson fit in the curriculum and instructional context? Is it the opening of a unit or a series of lessons?*	This lesson is the introductory lesson of a unit. The goal of the overall unit is to discover the Pythagorean Theorem, orthogonal systems, the practical applications of the Pythagorean Theorem, triangle ratios and proportions, irrational numbers and squares/square roots. This particular lesson is designed as the introduction to the unit. We will discuss the overall goals of the unit as we begin the first lesson.

	Prior Knowledge: The students should already have experience with the Cartesian Plane, locating points in the plane, measurement, calculating the areas of squares and working with square roots. The students should already be comfortable with the coordinate plane, squares and square roots. As a result this lesson will be utilized to reactivate and refresh that knowledge in preparation for this unit on the Pythagorean Theorem. I will also assess any weakness in these areas as these skills are necessary to accomplish this unit. If there is weakness in the areas of square roots, lesson two is ready to teach that skill. The student exploration will reveal the relationship that the sum of the sides of a triangle is larger than the hypotenuse. This will be revealed through "taxicab geometry" where students determine distances from one point to another while constrained to traveling along fixed horizontal and vertical paths (like roads). Students will conclude that the "fastest" path is a diagonal. Students will determine this through measurement. The triangle inequality theorem will be discussed in relation to this activity. The technology piece will be the use of an applet to summarize the triangle inequality theorem. This is designed as an introductory lesson for the unit and a review lesson to reactivate pre-requisite skills. It is planned for an 82-minute block. This group of students is behind in their math skills and is working on their algebra and geometry foundation skills in order to get ready for algebra. Many of the students have IEPs. As a result, both the eighth and tenth grade standards are listed above. The students still need to meet the eighth grade standards but the tenth grade standard is listed as another goal.
Opportunities to Learn *Differentiation: Materials, Learners and Environments*	**Plans to differentiate instruction:** Students who are struggling with the exploration will be supported in a scaffolded approach. Students who finish quickly will be posed an additional challenge as noted on the record sheet which is to see if any other triangle types have a similar property. Students who are visual will relate to the coordinate grid exploration. Visual: the triangle inequality applet Kinesthetic: plotting and measuring Verbal: partner work will make the experience more enjoyable. Describing their observations is verbal

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Accommodations and modifications:
Students with disabilities will work in groups. Peer partners can help with manipulating the square pieces.

Student X must be seated in the front of the class during discussions due to vision issues.

Environment factors:
Classroom space needed to allow students to work in groups of 2; desks will be moved together to allow groups to work together.
White board or black board space.
Computer and projector to display technology applet

Materials:
Exploration for in class activity
Pre-requisite skill assessment
Notebooks
Pencil
Ruler
Computer/laptop
Internet connection to access to:
1) Show a street map of Washington D.C.
http://dcpages.com/Tourism/Maps/Washington-DC-Street-Map.shtml

2) The following applet demonstrates this for any triangle.
http://www.mathopenref.com/triangleinequality.html
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<td>Objectives</td>
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Students will:
• activate prior knowledge about the area of a square, calculating square roots, coordinate plane and coordinate plane geometry
• create a table to organize observations
• draw conclusions from the data collected
• justify the observations
• explain/communicate the process used during the exploration process
• discover the relationship of the sides of a triangle and the triangle inequality theorem
•
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<td>Instructional Procedures</td>
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LAUNCH:

<u>Introduction:</u>
I will begin the class with and overall introduction to the unit sharing with the students the major objectives we want to accomplish within the next 2-3 weeks:
1) Coordinate plane refresh skills; and develop strategies for finding the distance between two points on a coordinate grid.
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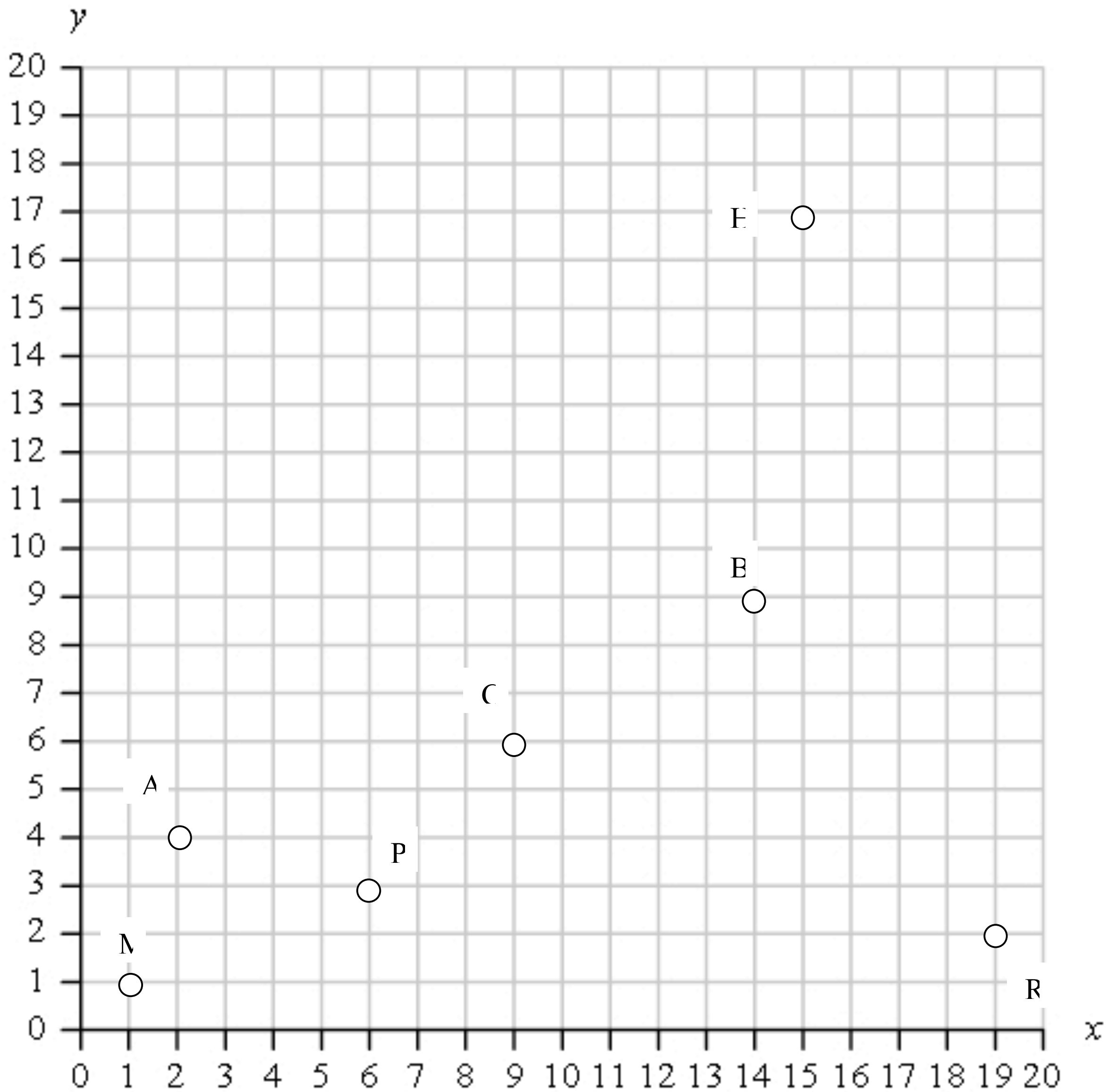
Holly Eaves	2) The knowledge of areas of squares and associated square roots will be refreshed. 3) There is a special property of right triangles we will explore. The property, called the Pythagorean Theorem was understood by the Egyptians many years before Pythagoras. A man named Pythagoras; a Greek who lived in the 6^{th} Century B.C. described the property in general mathematical language. This property has been verified and proved and is therefore called a theorem. 4) The Egyptians were very clever in applying what we call the Pythagorean Theorem in their everyday work in practical ways. We will also apply the Pythagorean Theorem to help us solve a number of everyday problems. 5) The concept of irrational numbers will be introduced The students will be told that they will be inducted into the Pythagorean Society if they complete the learning challenges within this unit. We can have a toga party, and they will be presented with certificates of their accomplishments. I will gauge the students' interest in the toga party. Ninth graders may not want to wear togas. <u>Assess a pre-requisite skill:</u> Next I will engage the class in student centered discussion with the intent to activate students' prior knowledge and determine students' mathematical language regarding their understanding of the area of a square and a square root with the following questions: We have discussed in the past the area of a square. Can somebody tell me the general properties of a square? How do you calculate the area of a square? What is a square root? How does that relate to the area of a square? Since this topic was covered by my Cooperating Teacher already, the hope is that this understanding is present and we can move more quickly into the unit. If there seems to be general understanding, the students will be given a quick assessment to check if the students have a grasp of these skills. If not, a more detailed exploration is planned in Lesson two. A pre-requisite skill assessment will be administered. <u>Setting the Context and Activating Prior Knowledge:</u> After the pre-requisite skill assessment is complete, we will begin the discussion of the Cartesian plane (coordinate grid). We will activate prior knowledge. Exploratory questions: Please plot the point described by the x,y coordinates of (4,6). Does the first number always represent the x axis? Does the second number always represent the number of spaces up or down on the y-axis? What is the relationship between the lines parallel to the x-axis and the line parallel to the y axis? Hopefully the students know the concept of perpendicular. The Cartesian plane is useful for many reasons but some cities are designed based on a coordinate grid philosophy for a primary reason that it is a good organizational structure where direction is implicit in the organization. The streets are straight to make maintenance easier. A

	good example of a city that is designed based on a coordinate grid is Washington D.C. Show a street map of Washington D.C. http://dcpages.com/Tourism/Maps/Washington-DC-Street-Map.shtml Establishing the mathematical language: I will be listening and encouraging the following key vocabulary: Coordinate grid, Cartesian Plane, coordinate pair, perpendicular, Auxiliary vocabulary: orthogonal Issuing the Challenge and setting expectations: Students will be given an exploration sheet. The exploration sheet will have a coordinate grid with various locations in a virtual city on it. Students will be told that they are training with taxi driver, Pythagoras. A passenger just got into the car and told you they were in a hurry. They needed to get from point A to point B fast. Disregarding speed what is the fastest street route you can travel? The students will be asked to record the distances traveling along the horizontal and vertical "streets" from point A to point B. Students will then be told that Pythagoras is frustrated and knows there is a more direct route. If they had special powers and did not have to travel the roads, how would you make Pythagoras proud by choosing the most direct route? If you had any means of getting from point A to point B, draw your fastest route and measure it. Compare this measurement to the route one would have to travel by taxicab. Please work with a partner. The lab sheet must be handed in, so please complete your exploration with a partner but I must receive a lab sheet from all students. Transition to Explore: Think about how a taxicab must travel. What would be the most direct route to any location if you did not have to travel the streets? What is a word that describes that direct route? Looking for the word diagonal. **EXPLORE:** Monitoring/ Observing and Promoting On Task Behavior The students will be given 15 minutes for the exploration. The table in the "Record Sheet for Exploration" below will be given as a scaffold rather than letting the students build the table on their own. This class needs the direction; otherwise time would be spent on helping them create the table as I have learned in the past. Asking Probing Questions What do you notice about the distance traveled on the roads even if you go up one and over one and try to travel a diagonal that way? What does it sum up to? What is the fastest route? What do you notice about the fastest route, the diagonal versus

	traveling in straight lines? What shape does this form? Can you generalize your observations in a mathematical way using mathematical symbols? <u>Issuing Extra Challenges</u> For students who finish the exploration phase quickly will be challenged to calculate the direct route without measurement. **SUMMARIZE/SHARE :** When the class comes back together for discussion, students will be asked to share their observations from the explore phase of the lesson using mathematical language. We will compile a table together as a class from the results that the class observed. Discussion will lead to the relationship that the sum of the sides of the triangle formed by the horizontal traverse, the vertical traverse, and the diagonal are always larger than the diagonal. This will lead us to talk about the triangle inequality theorem. <u>The triangle inequality theorem states that any side of a triangle is always shorter than the sum of the other two sides.</u> The following applet demonstrates this for any triangle. *http://www.mathopenref.com/triangleinequality.html* <u>Student Notebook Summary:</u> Students will put the Triangle Inequality Theorem in their notebooks with some examples. Students will be encouraged to take notes on the summarization of the exploration with the important points being: The Triangle Inequality Theorem with some examples Drawing of a right triangle from their coordinate geometry The properties of a right triangle <u>Closure:</u> Pythagoras would be proud of you all. You will have completed another Pythagorean challenge on your way to be inducted into the secret math society of the Pythagoreans. Understanding the coordinate grid and the right triangle geometries will be useful to you in your future as a Pythagorean.
Assessment	The students will be assessed on the pre-requisite skills for this unit regarding the areas of squares and evaluating square roots. The students will be informally assessed throughout the class with assessment coming from the questions to be asked during the exploration phase listed above.

	The student lab sheet will be turned in as a class work assessment.
Reflections *This section to be completed only if lesson plan is implemented.*	*Student Work Sample 1 – Approaching Proficiency:* *Student Work Sample 2 – Proficient:* *Student Work Sample 3 – Exceeds Proficiency:* *Lesson Implementation:*



B = Bank
M = Washington D.C. Mall
C = Starbucks
P = Pizza
A = Art Museum
R = River
H = Hotel

Record Sheet for Taxicab Geometry Exploration Phase:

Name:_______________________ Date:__________________

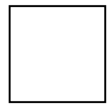
Point Leaving From	Point Going to	Distance traveled horizontally	Distance traveled vertically	Add the units from the previous two columns	Diagonal distance measured
B	C				
M	A				
A	P				
P	M				
M	R				
R	H				

Measure the distance traveled horizontally and vertically. Add up the distances traveled on the roads. Place that measurement in column 5.
Draw the line for the fastest route and measure it. Place this measurement in column 6.

What do you notice about the fastest route? How would you describe this route?

Pre-requisite Skill Assessment – Areas of squares and square roots

Name:________________________ Date:___________________

Show your work in the space provided:  3

1. Find the area of the square with a side length of 3 ________________.
2. Find the area of the square with a side length of 4.5 ________________.
3. Find the area of the square with a side length of 3 1/3 ________________.
4. Find the area of the square with a side length of 15 ________________.
5. Find the area of the square with a side length of 8 ________________.
6. Find the area of the square with a side length of $\sqrt{2}$ ________________.

Provide the square roots:

1. $\sqrt{9}$ = ______________
2. $\sqrt{100}$ = ______________
3. $\sqrt{2}$ = ______________
4. $\sqrt{64}$ = ______________
5. $\sqrt{256}$ = ______________
6. $\sqrt{16}$ = ______________
7. $\sqrt{85}$ = ______________
8. $-\sqrt{25}$ = ______________

Lesson 2 – Secrets of Squares & Square Roots

Grade/Content Area	For ninth grade math students in a pre-algebra (math foundations) class.
Lesson Title	***Pythagorean Challenge 2 – The Secrets of Squares and Square Roots***
State Standards: GLEs/GSEs **National Content Standards**:	RI Standards: M(N&O)–8–4 Accurately solves problems involving proportional reasoning (percent increase or decrease, interest rates, markups, or rates); multiplication or division of integers; and squares, cubes, and taking square or cube roots. (Local) M(CCR)–8–1 **Students will communicate their understanding of mathematics** and be able to: • Articulate ideas clearly and logically in both written and oral form. • Present, share, explain, and justify thinking with others and build upon the ideas of others to solve problems. • Use mathematical symbols and notation. • Formulate questions, conjectures, definitions, and generalizations about data, information, and problem situations.
Context of the Lesson *Where does this lesson fit in the curriculum and instructional context? Is it the opening of a unit or a series of lessons?*	This lesson is the second one in the unit. Lesson 1 will establish the need for a lesson on this skill. If it is determined that the students have mastery of squares and square roots, this lesson can be bypassed. This particular lesson establishes the pre-requisite skill of squares and square roots. Prior Knowledge: The students should be familiar with calculating the area of a square. The technology piece will be the use of calculators. It is planned for half of an 82 minute block.
Opportunities to Learn *Differentiation: Materials, Learners and*	**Plans to differentiate instruction:** Students who are struggling with the exploration will be supported in a scaffolded approach. Students who finish quickly will be posed an additional challenge to estimate the values of squares. Visual: http://www.brainpop.com/math/numbersandoperations/squareroots/preview.weml

Environments	Verbal: partner work will make the experience more enjoyable. Describing their observations is verbal **Accommodations and modifications:** Students with disabilities will work in groups. Peer partners can help with manipulating the square pieces. Student X must be seated in the front of the class during discussions due to vision issues. **Environment factors:** Classroom space needed to allow students to work in groups of 2; desks will be moved together to allow groups to work together. White board or black board space. Computer and projector to display technology applet **Materials:** Graph paper Notebooks Pencil Ruler Computer/laptop Internet connection to access to: 1) BrainPop square root review http://www.brainpop.com/math/numbersandoperations/squareroots/preview.weml
Objectives	Students will: • Understand the relationship between squares and square roots •
Instructional Procedures	**LAUNCH:** Setting the Context and Activating Prior Knowledge: A student centered discussion will be lead to assess prior knowledge and the level of mastery of the students. Students will be asked 'What number squared is 49?" We will discuss answers and seek student consensus. Then the question will be asked, "What is the square root of 49?" I will ask the students what they think square "root" means. After discussion, we will summarize with a tree analogy. That a root is like going down to the beginning; like a tree root. The tree's health begins at the roots. We will discuss another example. What is the square root of 100? When the square root of a square is an integer, it is called a perfect square. Look at the diagonal of the multiplication chart on the wall what do you notice about the perfect squares on the chart? Students should respond that they are on the diagonal of the chart. That may help you form perfect squares. Establishing the mathematical language:

	I will be listening and encouraging the following key vocabulary: Square, square root, perfect square, integer, Auxiliary vocabulary: radical <u>Issuing the Challenge and setting expectations:</u> Students will be given grid paper. Students will be issued another Pythagorean challenge. Students will draw squares of different sizes on their grid papers and find the square roots of the areas of square. When the square root of a square is an integer, it is called a perfect square. Draw perfect squares and discover the secret of square roots. <u>Transition to Explore:</u> Review the area of a square as s x s = s^2 = area of a square. Example: I can find the square root of 16 by using the area of a square. Draw a square with area 16. Verify the area by counting the units within the square. I have just drawn a perfect square. The square root symbol is drawn as √. This is called a radical sign. Count the number of units on along the side of the square. What is the measure of the side of this square? Students answer 4. Now ask what is the square root of 16? The answer is 4. Connection is made. **EXPLORE:** <u>Monitoring/ Observing and Promoting On Task Behavior</u> The students will be given 10 minutes for the exploration. The students will work individually since the activity is so short. Students will be monitored for understanding <u>Asking Probing Questions</u> What do you notice about the relationship of the square root and the area of the square? Can you draw a perfect square with area 24? Can you draw a perfect square with area 66? <u>Issuing Extra Challenges</u> Can the square root of a square be a real number, something other than an integer? How would you draw that on your grid paper? What would you estimate the square root to be? **SUMMARIZE/SHARE :** When the class comes back together for discussion, we will look at the perfect squares the students discovered and check the answers. Then we will address the probing can you draw a square with area 24? We will eventually come to the fact that 24 lies between 25 and 16, so the square root of 24 should be between 4 and 5. <u>Student Notebook Summary:</u> s x s = s^2 = area of a square √ is called a radical

	The square root of a perfect square is an integer. Example √49 = 7 The radical sign was originally an "r" for root. <u>Closure:</u> Pythagoras would be proud of you all. You will have completed another Pythagorean challenge on your way to be inducted into the secret math society of the Pythagoreans. Understanding squares and square roots will be useful to you as a Pythagorean inductee.
Assessment	The students will be informally assessed throughout the class with assessment coming from the questions to be asked during the exploration phase listed above. The attached homework will be handed in for assessment.
Reflections *This section to be completed only if lesson plan is implemented.*	*Student Work Sample 1 – Approaching Proficiency:* *Student Work Sample 2 – Proficient:* *Student Work Sample 3 – Exceeds Proficiency:* *Lesson Implementation:*

Name: ______________________________

Looking for Square Roots

Find the square root of each perfect square.

1. $\sqrt{100}$ __________
2. $\sqrt{225}$ __________
3. $\sqrt{256}$ __________
4. $\sqrt{625}$ __________
5. $\sqrt{144}$ __________

The following numbers are *not* perfect squares. Which two integers does the square root lie between?

6. $\sqrt{13}$ __________
7. $\sqrt{22}$ __________
8. $\sqrt{32}$ __________
9. $\sqrt{108}$ __________
10. $\sqrt{52}$ __________

Determine if each number is a perfect square. Explain your answer.

11. 289 ____

____

12. 645 ____

____

Standard 1 / Activity 4
Indiana Mathematics Grade 7 Curriculum Framework, October 2002

Black Line Master 1
page 39

Worksheet from Lesson Planet website retrieved 11/28/09
http://www.indianastandardsresources.org/files/math/math_7_1_6.pdf

Looking for Square Roots

Teacher Directions

Distribute one copy of the BLM *Looking for Square Roots* to each student. Ask students to answer each question and check the answer by squaring the square roots. Allow students to use their graph paper to find the square roots.

Answer Key

1.) 10

2.) 15

3.) 16

4.) 25

5.) 12

6.) 3 and 4

7.) 4 and 5

8.) 5 and 6

9.) 10 and 11

10.) 7 and 8

11.) 289 is a perfect square. A square can be drawn with area 289 square units with the length of the sides 17 units. 17 is an integer value, therefore 289 is a perfect square.

12.) 645 is not a perfect square. A square with an area of 645 square units would have sides that measure somewhere between 25 and 26.

Answer Sheet from Lesson Planet website retrieved 11/28/09
http://www.indianastandardsresources.org/files/math/math_7_1_6.pdf

Lesson 3 - Pythagoras and the magical property of right triangles

<table>
<tr><td>Grade/Content Area</td><td>For ninth grade math students in a pre-algebra (math foundations) class.</td></tr>
<tr><td>Lesson Title</td><td>Pythagorean Challenge 3 - Pythagoras and the magical property of right triangles.</td></tr>
<tr><td>State Standards:
GLEs/GSEs
National Content Standards:</td><td>RI Standards:

M(G&M)–8–2 <u>Applies the Pythagorean Theorem</u> <u>to find a missing side of a right triangle, or in problem solving situations</u>. (Local)

M(G&M)–10–2 Makes and defends conjectures, constructs geometric arguments, uses geometric properties, or uses theorems to solve problems involving angles, lines, polygons, circles, or right triangle ratios (sine, cosine, tangent) within mathematics or across disciplines or contexts (e.g., Pythagorean Theorem, Triangle Inequality Theorem). (State)

M(N&O)–8–6 Uses a variety of mental computation strategies to solve problems (e.g., using compatible numbers, applying properties of operations, using mental imagery, using patterns) and to determine the reasonableness of answers; and mentally calculates benchmark perfect squares and related square roots (e.g., 1^2, 2^2, … , 12^2, 15^2, 20^2, 25^2, 100^2, 1000^2); (Local)

M(CCR)–8–1 Students will communicate their understanding of mathematics and be able to:
Articulate ideas clearly and logically in both written and oral form.
Present, share, explain, and justify thinking with others and build upon the ideas of others to solve problems.
Use mathematical symbols and notation.
Formulate questions, conjectures, definitions, and generalizations about data, information, and problem situations.
NCTM Communication Standard
communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
analyze and evaluate the mathematical thinking and strategies of others;
NCTM Representation Standard
Create and use representations to organize, record, and communicate mathematical ideas.</td></tr>
<tr><td>Context of the Lesson
Where does this lesson fit in the curriculum and instructional</td><td>This lesson is the third lesson of the unit on the Pythagorean Theorem.

This particular lesson will provide the entry point for the students’ discovery of the Pythagorean Theorem. Students</td></tr>
</table>

context? Is it the opening of a unit or a series of lessons?	should already have knowledge of two dimensional shapes, knowledge of area, and the various methods for determining the area of a shape, specifically a square. Students should know how to square a number and take the square root. The student exploration will reveal the relationship between the areas of squares and the sides of a right triangle utilizing squares of different areas made out of paper. These manipulatives will be enough for the student exploration. (I would like to credit Cornelius DeGroot as a resource for this lesson as he demonstrated this activity in class) The technology piece is the utilization of an applet on the NCTM Illuminations website that will allow us to work with squares of varying areas even those that are not of integral measurement. Further exploration will be done in the summary phase of the lesson. Another applet will be used to increase the student's interest for the next lesson. This is designed as a one day lesson within an 82 minute block.
Opportunities to Learn *Differentiation: Materials, Learners and Environments*	**Plans to differentiate instruction:** Students who are struggling with the exploration will be supported in a scaffolded approach. Students who finish quickly will be posed an additional challenge as noted on the record sheet which is to see if any other triangle types have a similar property. Students who are visual will relate to the geometric proof introduction. The exploration has a kinesthetic aspect. **Accommodations and modifications:** Students with disabilities will work in groups. Peer partners can help with manipulating the square pieces. Student X must be seated in the front of the class due to vision issues. **Environment factors:** Classroom space needed to allow students to work in groups of 2; desks will be moved together to allow groups to work together. White board or black board space. Computer and projector to display technology applet **Materials:** Envelopes containing squares with grids on them of different areas (see Dynamic Paper attached) The Record Sheet Pencil Ruler Computer/laptop

	Internet connection to access: http://illuminations.nctm.org/ActivityDetail.aspx?ID=164 http://www.brainpop.com/math/geometryandmeasurement/pythagoreantheorem/preview.weml
Objectives	Students will: • Discover the relationship between the areas of squares that form a right triangle • Develop an introductory understanding of geometric proof • Activate prior knowledge about the area of a square, triangle classifications • Classify, illustrate and explain triangle types with focus on right triangles • Apply the Pythagorean Theorem in determining the hypotenuse (the lesson will be extended to find the missing side and applications) • Create a table to organize observations • Draw conclusions from the data collected • Justify the observations • Explain/communicate the process used during the exploration process
Instructional Procedures	**LAUNCH:** Setting the Context and Activating Prior Knowledge: I will begin the class with student centered discussion with the intent to activate students' prior knowledge and determine students' mathematical language regarding triangles specifically right triangles, the area of a square and the area of a triangle with the following questions: We have discussed, in the recent past, what an area is? Who can explain area? Who can demonstrate how we obtain the area of a square? What types of triangles can you list? Let us discuss what you know about the properties of these triangles. Demonstrate the area of a triangle. How would you define a right angle and therefore a right triangle? Are there right angles, triangles in a square or rectangle? (15 minutes maximum) Establishing the mathematical language: I will be listening and encouraging the following key vocabulary: Right triangle, isosceles triangle, equilateral triangle, acute triangle, obtuse triangle, scalene triangle, square, area, hypotenuse, legs or sides of a triangle, diagonal, Pythagorean theorem. Auxiliary vocabulary: three sided polygon, perimeter Issuing the Challenge and setting expectations: Each group of two students has been given an envelope. Inside the envelope, there are squares of various areas (These will be cut out of the paper shown below on the Dynamic paper). Use these squares to form

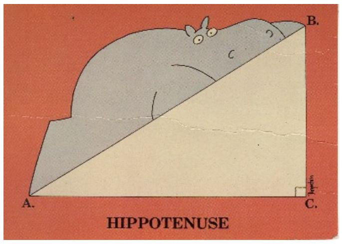
	right triangles. Record the areas of the squares that form the two legs and the hypotenuse in a table. Discuss with your partner. Draw conclusions about what you discover to discuss when we come back together as a class. <u>Transition to Explore:</u> Definition of a Right triangle: A triangle having a right angle. One of the angles of the triangle measures 90 degrees. The side opposite the right angle is called the hypotenuse. The two sides that form the right angle are called the legs. A right triangle has a special property that we are going to explore. **EXPLORE:** <u>Monitoring/ Observing and Promoting On Task Behavior</u> The students will be given 15 minutes for the exploration. The table in the "Record Sheet for Exploration" below will be given as a scaffold rather than letting the students build the table on their own. This class needs the direction; otherwise time would be spent on helping them create the table as I have learned in the past. <u>Asking Probing Questions</u> Can you make a right triangle where a leg of the triangle is longer than the hypotenuse? Can the areas of the squares on the legs be larger than the hypotenuse? Will this be a right triangle? Compare the areas of the squares on the legs to that on the hypotenuse? Can you generalize your observations in a mathematical way? <u>Issuing Extra Challenges</u> Direction was given on the "Record Sheet for Exploration" as a further challenge to experiment with the relationships of the squares in forming other types of triangles. For students who finish the exploration phase quickly, this will allow some differentiated instruction. **SUMMARIZE/SHARE :** (25 minutes) When the class comes back together for discussion, students will be asked to share their observations from the explore phase of the lesson using mathematical language. We will compile a table together as a class from the results that the class observed. Discussion will lead to the relationships of the areas of the squares that form the right triangle. Hopefully the students will notice that the sum of the areas of the squares on the legs of the triangle will

	equal the area of the square on the hypotenuse. Since the exploration phase worked with physical manipulatives that created right triangles with sides of integral length of smaller sizes, the following Pythagorean Review Applet for the squares will be utilized for the class to discover the Pythagorean Theorem further. http://illuminations.nctm.org/ActivityDetail.aspx?ID=164 This applet does the same thing the students did in the explore phase but allows students to see that the areas of the squares could have a non-integral lengths. They will also be able to see how the relationships of the squares change in real time. This can be projected on the white board for the class to see. Students can add data to their tables from this applet. When the students understand, the following BrainPop video will be used as a summary (or it can be used as a refresher on a subsequent day). http://www.brainpop.com/math/geometryandmeasurement/pythagoreantheorem/preview.weml Students will put their table of data collected in their notebooks along with a right triangle formed by the squares. Students will be encouraged to take notes on the summarization of the exploration with the important points being: Drawing of a right triangle The properties of a right triangle Pythagorean Theorem $A^2 + B^2 = C^2$ *It will be emphasized that this relationship holds for any similar areas on the sides of a right triangle.* Depending on time, we may say that we came to the Pythagorean Theorem by looking at numbers and forming a conjecture. We can support our conjecture by finding more examples that work for the Pythagorean Theorem. Another way to prove our Theorem is through geometric proof. What is a geometric proof? Students will be asked to contribute their thoughts on what they think a geometric proof is. This will be used as a lead into the next day's lesson. Geometric proofs of the Pythagorean Theorem will be demonstrated. The following visual proofs will be shown and discussed. Pythagorean Theorem Animated Proof http://www.mathopenref.com/pythagorasproof.html

	Proof without words Pythagorean Theorem http://illuminations.nctm.org/ActivityDetail.aspx?ID=30 Students will be encouraged to try out the following virtual manipulative website on their own if they have a computer resource at home. Virtual Manipulative for Pythagorean Theorem: http://nlvm.usu.edu/en/nav/frames_asid_164_g_3_t_3.html?open=instructions&from=category_g_3_t_3.html Closure: Pythagoras would be proud of you all. You will have completed another Pythagorean challenge on your way to be inducted into the secret math society of the Pythagoreans. Understanding the Pythagorean Theorem and the right triangle geometries will be useful to you in your future.
Assessment	The students will be informally assessed throughout the class. The following probing questions will be helpful: Is the square on the hypotenuse larger or smaller than the square or squares on the legs of the triangle? Is there any relationship between the area of the right triangle and the area of the squares? How does the area of the squares relate to the right triangle? The student lab sheet will be handed in at the end of class as a formative assessment. Students will be assigned a homework worksheet, exercising the basics of finding the length of a hypotenuse of a right triangle.
Reflections *This section to be completed only if lesson plan is implemented.*	*Student Work Sample 1 – Approaching Proficiency:* *Student Work Sample 2 – Proficient:* *Student Work Sample 3 – Exceeds Proficiency:* *Lesson Implementation:*

Record Sheet for Exploration Phase:

Name:________________________ Date:___________________



Length of Leg 1	Length of Leg 2	Length of Hypotenuse	Area of Square on Leg 1	Area of Square on Leg 2	Area of the square on the hypotenuse

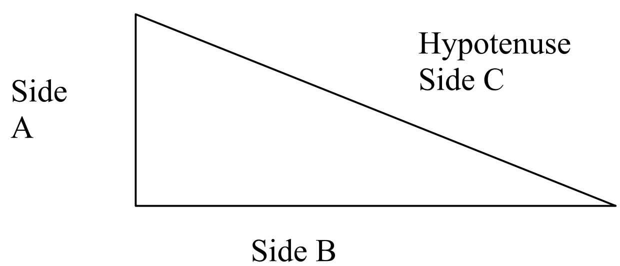
Write down any observations here as to the relationship observed in the table above:

If you have extra time, can you form other triangles that are not right triangles and note any other relationships between the areas of the squares forming the sides of the triangle?
Record your observations below with pictures.

Pythagorean Theorem Practice:

Name:________________________   Date:___________________

Given the two sides of the right triangle, what is the hypotenuse? Show your work.
$A^2 + B^2 = C^2$



1. Side A is 3 inches
   Side B is 4 inches
   What is the length of the hypotenuse? ______________

2. Side A is 6 inches
   Side B is 8 inches
   What is the length of the hypotenuse? ______________

3. Side A is 2.5 cm
   Side B is 4.5 cm
   What is the length of the hypotenuse? ______________

4. Side A is 2.5 inches
   Side B is 4.5 inches
   What is the length of the hypotenuse? ______________

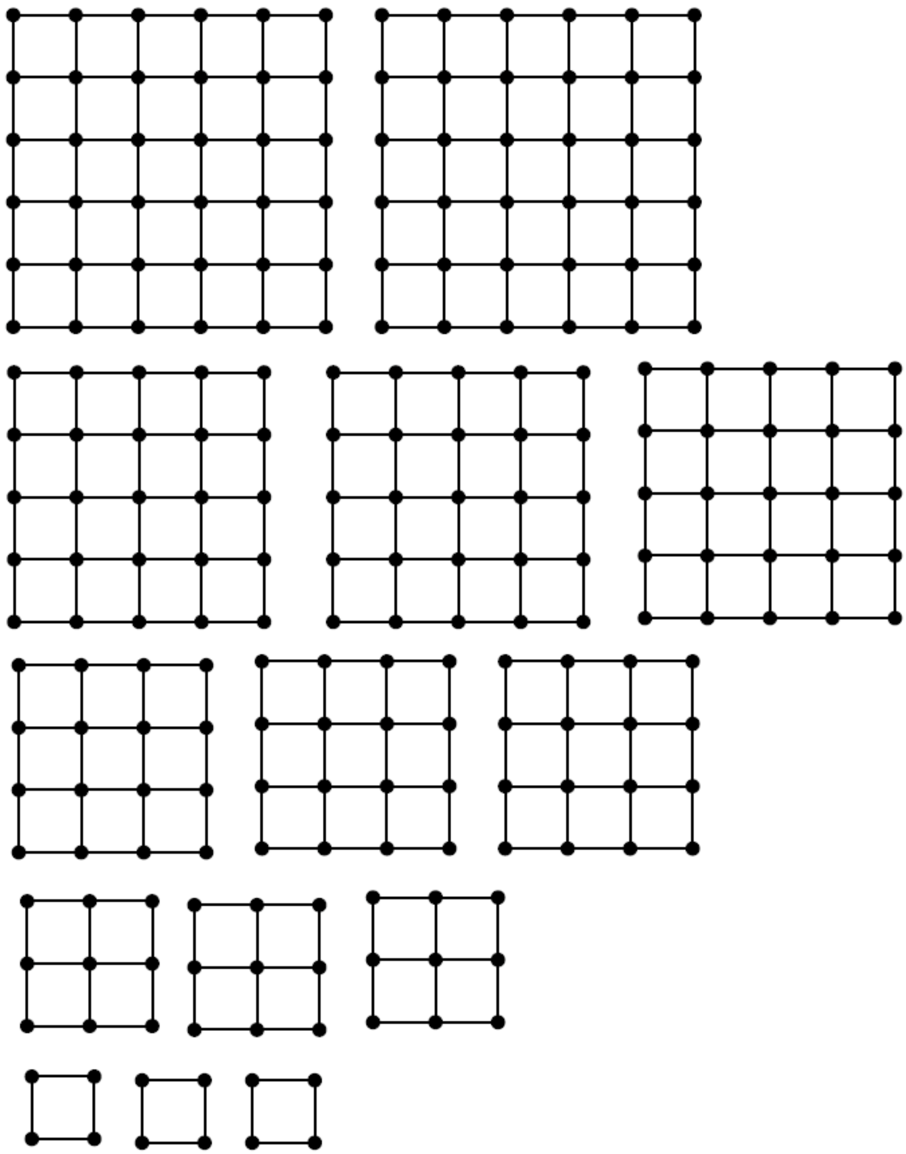
5. Side A is 10 cm
   Side B is 10 cm
   What is the length of the hypotenuse? ______________

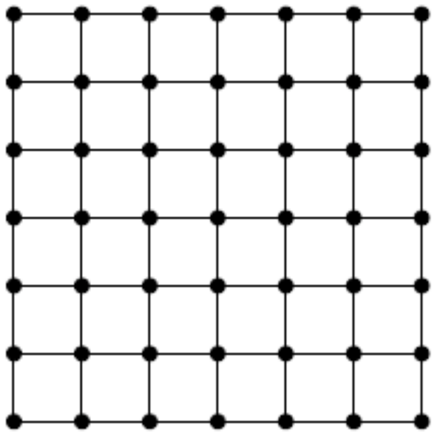
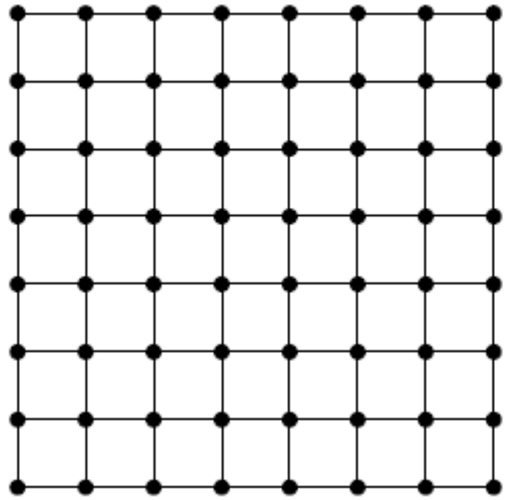
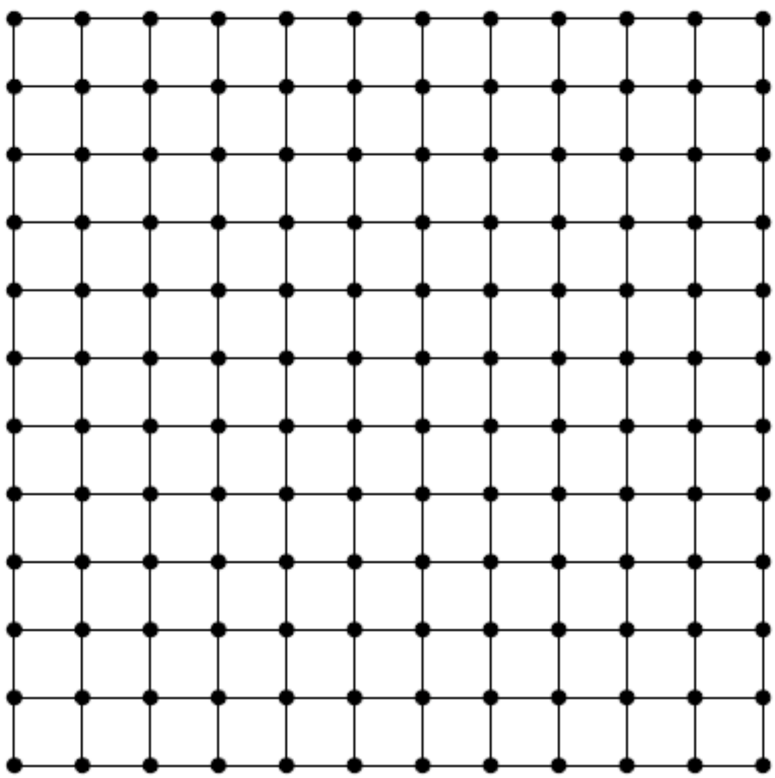
6. Side A is 8 cm
   Side B is 12cm
   What is the length of the hypotenuse? ______________

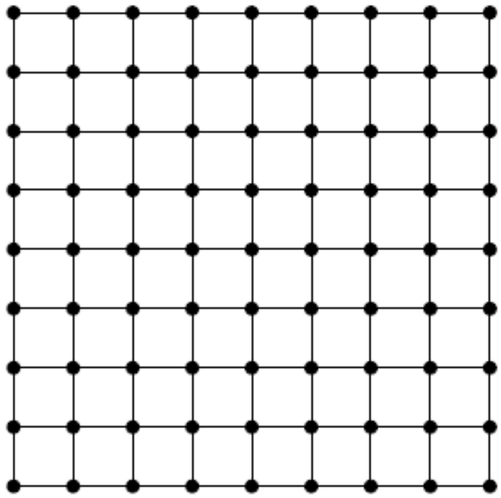
7. Side A is 8 inches
   Side B is 5 inches
   What is the length of the hypotenuse? ______________

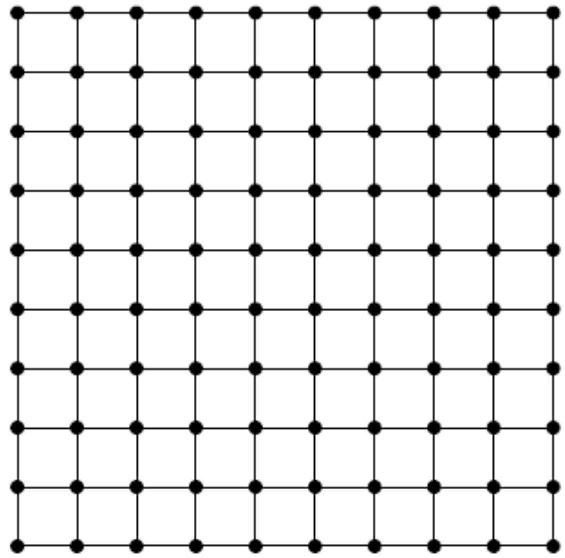
Dynamic Paper

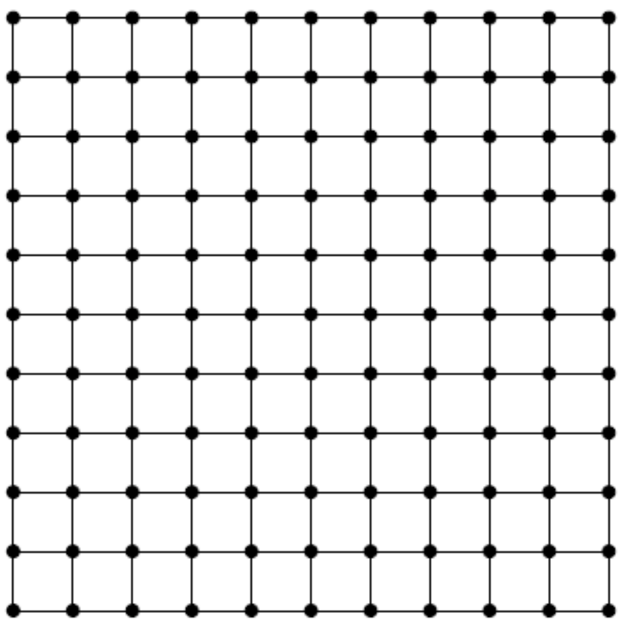
NAME ________________________









Lesson 4 – Acute and Obtuse Triangles, Pythagorean Triples

Grade/Content Area	For ninth grade math students in a pre-algebra (math foundations) class.
Lesson Title	*Acute and Obtuse Triangles* *Pythagorean Triples*
State Standards: GLEs/GSEs **National Content Standards**:	RI Standards: M(G&M)–10–2 **Makes and defends conjectures, constructs geometric arguments, uses geometric properties, or uses theorems to solve problems** involving angles, lines, polygons, circles, or right triangle ratios (sine, cosine, tangent) within mathematics or across disciplines or contexts (e.g., **Pythagorean Theorem, Triangle Inequality Theorem**). (State) M(N&O)–8–4 Accurately solves problems involving proportional reasoning (percent increase or decrease, interest rates, markups, or rates); multiplication or division of integers; and squares, cubes, and taking square or cube roots. (Local) M(CCR)–8–1 **Students will communicate their understanding of mathematics** and be able to: • Articulate ideas clearly and logically in both written and oral form. • Present, share, explain, and justify thinking with others and build upon the ideas of others to solve problems. • Use mathematical symbols and notation. • Formulate questions, conjectures, definitions, and generalizations about data, information, and problem situations. NCTM Communication Standard • communicate their mathematical thinking coherently and clearly to peers, teachers, and others; • analyze and evaluate the mathematical thinking and strategies of others;
Context of the Lesson *Where does this lesson fit in the curriculum and instructional context? Is it the opening of a unit or a series of lessons?*	This lesson is an extension of lesson three of this unit. Lesson three focused on getting the students to recognize the special property of right triangles, the Pythagorean Theorem. Lesson four is designed to emphasize two concepts that are natural extensions of lesson three; 1) Bring the student's attention to the other classifications of triangles; the acute and obtuse triangles and how the relationship $a^2 + b^2 >$ or $< c^2$ can indicate the difference. 2) Emphasize the Pythagorean triples that were discovered in lesson three. Prior Knowledge:

	Knowledge from lesson three where the students used squares to form the sides of right triangles. They learned the Pythagorean theorem as a result of that exploration. Students will also need to reference their table generated during the previous lesson in order to find Pythagorean triples. The technology piece will be the use of an applet to demonstrate the change in the relationship of the sides as a triangle is changed from acute to right to obtuse. It is planned for an 82 minute block.
Opportunities to Learn *Differentiation: Materials, Learners and Environments*	**Plans to differentiate instruction:** Students who are struggling with the exploration will be supported in a scaffolded approach. Students who are visual will relate to the exploration. Visual: the applet Kinesthetic: working with squares to make triangles, manipulatives. Verbal: partner work will make the experience more enjoyable. Describing their observations and justifying their conclusions is verbal **Accommodations and modifications:** Students with disabilities will work in groups. Peer partners can help with manipulating the square pieces. Student X must be seated in the front of the class during discussions due to vision issues. **Environment factors:** Classroom space needed to allow students to work in groups of 2; desks will be moved together to allow groups to work together. White board or black board space. Computer and projector to display technology applet **Materials:** Exploration for in class activity Notebooks Pencil Squares of varying areas Computer/laptop Internet connection to access to: 1) http://www.mathopenref.com/acutetriangle.html demonstrates the difference between an acute, obtuse and right triangle. 2) http://www.mathopenref.com/triangleclassify.html

Objectives	Students will: • Classify triangles (acute, obtuse and right) through the $a^2 + b^{2\,(>,\,<,\ \text{or}\ =)}\, c^2$ relation. • Understand and utilize Pythagorean triples • Create a table to organize observations • Draw conclusions from the data collected • Justify the observations • Explain/communicate the process used during the exploration process
Instructional Procedures	**LAUNCH:** Setting the Context and Activating Prior Knowledge: I will begin the class with student centered discussion with the intent to activate students' prior knowledge, remind them of the lesson from the previous day. First we will discuss the concept of Pythagorean Triples defined as: Let a, b, c be sides of a right triangle, where a and b are the sides or legs of the right triangle and c is the hypotenuse. The Pythagorean Theorem is written in equation form as: $a^2 + b^2 = c^2$. If a, b, and c are all integers then the set of the three integers fulfilling this equation is called a Pythagorean Triple. The students will take notes on this definition. It will be emphasized with the following page: http://www.mathsisfun.com/numbers/pythagorean-triples.html Students will be asked to take out their lab sheets from lesson three and see if they can list some Pythagorean triples discovered the previous day. They should have a few of them. The students will also have some real number triples that we discovered the previous day with the applet. The students will be looking for the integer triples. As a class, we will form a list of Pythagorean triples. We should have at least found: (3,4,5) (5,12,13) (6,8,10) (9,12,15) The smallest Pythagorean Triple is (3,4,5). I will also show then a list of triples online at the same "mathisfun" website referenced above. These will be recorded in their notebooks and will be referenced again later when we discuss right triangle ratios and when we apply the Pythagorean Theorem. At this time, I will begin discussion of obtuse and acute triangles. We will then perform the same activity in lesson three; forming triangles with squares, only now we will focus on creating triangles that are either obtuse or acute. Establishing the mathematical language: I will be listening and encouraging the following key vocabulary: Right triangle, Pythagorean Theorem, Pythagorean Triples hypotenuse, legs or sides of a triangle, diagonal, areas of squares, obtuse, acute Auxiliary vocabulary: none

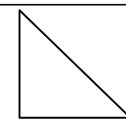
	Issuing the Challenge and setting expectations: Each group of two students has been given an envelope. Inside the envelope, there are squares of various areas (These will be cut out of the paper shown below on the Dynamic paper). Use these squares to form obtuse and acute triangles. Record the areas of the squares that form the two legs and the hypotenuse in a table. Discuss with your partner. Draw conclusions about what you discover to discuss when we come back together as a class. Transition to Explore: Definition of an acute triangle: A triangle having all three interior angles less than 90 degrees. Definition of an obtuse triangle: A triangle having one interior angle greater than 90 degrees. Reminder: A right triangle has a special property that we are going to explore. **EXPLORE:** Monitoring/ Observing and Promoting On Task Behavior The students will be given 15 minutes for the exploration. The table in the "Record Sheet for Exploration – Acute, right or Obtuse?" below will be given as a scaffold rather than letting the students build the table on their own. This class needs the direction; otherwise time would be spent on helping them create the table as I have learned in the past. Asking Probing Questions Which sides are the legs? In a right triangle the longest side is the hypotenuse; do we have a name for the longest side in an acute or obtuse triangle? How do you know you have an interior angle that is greater than 90 degrees? Less than 90 degrees? Can you generalize your observations in a mathematical way? Issuing Extra Challenges For students who finish quickly (for differentiation), they will be challenged to attempt to create the Pythagorean triple right triangles that we discussed earlier. If they do not have squares of the right size they will be encouraged to make a square the size they need. **SUMMARIZE/SHARE :** When the class comes back together for discussion, students will be asked to share their observations from the explore phase of the lesson using mathematical language. We will compile a table together as a class from the results that the class observed.

	The class should find the following relationships: Right Triangle $a^2 + b^2 = c^2$ Obtuse $a^2 + b^2 < c^2$ Acute $a^2 + b^2 > c^2$ Student Notebook Summary: www.brainpop.com "Types of Triangles" The following website displays the classifications http://www.mathopenref.com/triangleclassify.html Classification by interior angles A triangle where one of the internal angles is obtuse (greater than 90 degrees). A triangle where all three internal angles are acute (less than 90 degrees). A triangle where one of its interior angles is a right angle (90 degrees). Equiangular – all interior angles are equal Right Triangle $a^2 + b^2 = c^2$ Obtuse $a^2 + b^2 < c^2$ Acute $a^2 + b^2 > c^2$ Pythagorean Triples: 3:4:5 6:8:10 9:12:15 5:12:13 8:15:17 Is $1:1:\sqrt{2}$ a Pythagorean triple? No because the $\sqrt{2}$ is not an integer. (foreshadowing) In knowing the 3:4:5 triple, one can utilize proportions to determine other Pythagorean triples through scaling proportions. This will be utilized in a future lesson. Closure: Pythagoras would be proud of you all. You will have completed another Pythagorean challenge on your way to be inducted into the secret math society of the Pythagoreans. Understanding the Pythagorean triples and triangles other than those of right triangles will be useful to you in your future.
Assessment	The students will be informally assessed throughout the class with assessment coming from the questions to be asked during the exploration phase listed above.

	The student lab sheet will be turned in as a class work assessment. A homework sheet will be given.
Reflections *This section to be completed only if lesson plan is implemented.*	*Student Work Sample 1 – Approaching Proficiency:* *Student Work Sample 2 – Proficient:* *Student Work Sample 3 – Exceeds Proficiency:* *Lesson Implementation:*

Record Sheet for Exploration: Acute, Right or Obtuse?

Name:________________________ Date:___________________

Area of square on Leg 1	Area of Square on Leg 2	Area of Square on longest side	Length of Leg 1	Length of Leg 2	Length of Longest side	$A^2 + B^2$ __ C^2 = > <
9	16	25	3	4	5	$A^2 + B^2 = C^2$ Right triangle 

Write down any observations here as to the relationship observed in the table above:

Pythagorean Triples Practice:

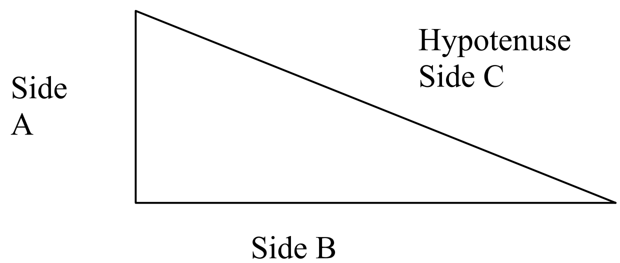
Name:________________________ Date:___________________

Given the triple (A, B, C), indicate what type of triangle is indicated? Show your work.

$A^2 + B^2 = C^2$ a right triangle.

$A^2 + B^2 > C^2$ an acute triangle

$A^2 + B^2 < C^2$ an obtuse triangle



8. Given the triple (5, 12, 13). What type of triangle is represented by this triple?

   ____

   25 + 144 = 169 Right triangle

9. Given the triple (4, 4, 4). What type of triangle is represented by this triple?

   ____

   16 + 16 > 16 Acute triangle (equilateral and equiangular)

10. Given the triple (2, 6, 7). What type of triangle is represented by this triple?

    ____

    4 + 36 < 49 Obtuse

11. Given the triple (3, 4, 6). What type of triangle is represented by this triple?

    ____

    9 + 16 < 36 Obtuse

12. Given the triple (4, 5, 6). What type of triangle is represented by this triple?

    ____

    16 + 25 > 36 Acute

Lesson 5 – Going the Distance

Grade/Content Area	For ninth grade math students in a pre-algebra (math foundations) class.
Lesson Title	*Pythagorean Challenge 5 – Going the distance*
State Standards: GLEs/GSEs **National Content Standards**:	RI Standards: M(G&M)–8–2 Applies the Pythagorean Theorem to find a missing side of a right triangle, or in problem solving situations. (Local) M(N&O)–8–4 Accurately solves problems involving proportional reasoning (percent increase or decrease, interest rates, markups, or rates); multiplication or division of integers; and squares, cubes, and taking square or cube roots. (Local) M(G&M)–10–2 **Makes and defends conjectures, constructs geometric arguments, uses geometric properties, or uses theorems to solve problems** involving angles, lines, polygons, circles, or right triangle ratios (sine, cosine, tangent) within mathematics or across disciplines or contexts (e.g., **Pythagorean Theorem, Triangle Inequality Theorem**). (State) M(G&M)–10–9 Solves problems on and off the coordinate plane involving distance, midpoint, perpendicular and parallel lines, or slope. (State) M(CCR)–8–1 **Students will communicate their understanding of mathematics** and be able to: • Articulate ideas clearly and logically in both written and oral form. • Present, share, explain, and justify thinking with others and build upon the ideas of others to solve problems. • Use mathematical symbols and notation. • Formulate questions, conjectures, definitions, and generalizations about data, information, and problem situations. NCTM Communication Standard • communicate their mathematical thinking coherently and clearly to peers, teachers, and others; • analyze and evaluate the mathematical thinking and strategies of others;
Context of the Lesson *Where does this lesson fit in the curriculum and instructional context? Is it the opening of a unit or a series of*	This lesson will utilize the student's knowledge of the Pythagorean Theorem to determine distances on the coordinate grid. Student strategies will be observed. The distance formula will be derived utilizing the Pythagorean Theorem rather than determining lengths from the grid. Prior Knowledge: The students should already have experience with the Cartesian

lessons?	Plane, locating points in the plane, calculating the areas of squares and working with square roots, using the Pythagorean Theorem. A student exploration similar to that utilized in Lesson 1 will be used. Pythagoras is again the taxi driver. Since Pythagoras has come back in time to drive a taxicab in our grid city, he is going to surprise his customers with his taxicab that can take the fastest route. The students have to calculate the fastest route without measuring this time. They have to use the Pythagorean Theorem to calculate the distance between points. This will impress Pythagoras. The Pythagorean Theorem BrainPop video will be used to refresh the students' memory on the Pythagorean Theorem. It is planned for an 82 minute block...
Opportunities to Learn *Differentiation: Materials, Learners and Environments*	**Plans to differentiate instruction:** Students who are struggling with the exploration will be supported in a scaffolded approach. Students who finish quickly will be posed an additional challenge as noted on the record sheet which is to see if any other triangle types have a similar property. Visual: the BrainPop video, coordinate grid exploration Verbal: partner work will make the experience more enjoyable. Describing their observations is verbal **Accommodations and modifications:** Students with disabilities will work in groups. Peer partners can help with manipulating the square pieces. Student X must be seated in the front of the class during discussions due to vision issues. **Environment factors:** Classroom space needed to allow students to work in groups of 2; desks will be moved together to allow groups to work together. White board or black board space. Computer and projector to display the video **Materials:** Exploration for in class activity Notebooks Pencil Ruler Computer/laptop Internet connection to access to:

	1) Pythagorean Theorem video clip on BrainPop http://www.brainpop.com/math/geometryandmeasurement/pythagoreantheorem/
Objectives	Students will: • apply the Pythagorean Theorem • derive the distance formula • calculate the shortest distance between two points on the coordinate grid •
Instructional Procedures	**LAUNCH:** Setting the Context and Activating Prior Knowledge: The Cartesian plane is useful for many reasons but some cities are designed based on a coordinate grid philosophy for a primary reason that it is a good organizational structure where direction is implicit. Every location has a description as a coordinate pair. A GPS utilizes a global positioning system similar in nature to the coordinate grid. In our virtual city of Washington D.C., Pythagoras is back as a taxicab driver. He does not have to drive on roads. He can get you there fast because he knows the shortest route between two points is a straight line. This is also called a hypotenuse of a right triangle formed by the points on our coordinate plane. Let's refresh our memory on the Pythagorean Theorem: http://www.brainpop.com/math/geometryandmeasurement/pythagoreantheorem/ Establishing the mathematical language: I will be listening and encouraging the following key vocabulary: Coordinate grid, Cartesian Plane, coordinate pair, perpendicular, Auxiliary vocabulary: orthogonal Issuing the Challenge and setting expectations: Students will be given an exploration sheet. The exploration sheet will have a coordinate grid with various locations in a virtual city on it. Students will be told that they are training with taxi driver, Pythagoras. Make Pythagoras proud and calculate the fastest route between two points using the Pythagorean Theorem. Please work with a partner. The lab sheet must be handed in, so please complete your exploration with a partner but I must receive a lab sheet from all students. Transition to Explore: An example will be done on the board; demonstrating how the students can get the measurements of the legs of the right triangle. They can then calculate the hypotenuse or fastest route between points.

EXPLORE:

Monitoring/ Observing and Promoting On Task Behavior

The students will be given 15 minutes for the exploration. The table in the "Record Sheet for Exploration" below will be given as a scaffold rather than letting the students build the table on their own. This class needs the direction; otherwise time would be spent on helping them create the table as I have learned in the past.

Asking Probing Questions

What shape is a good shape to try to draw when you are calculating a diagonal?
Can you think of another method to calculate the distance between two points without drawing a right triangle?
Can you generalize your observations in a mathematical way using mathematical symbols?

Issuing Extra Challenges

For students who finish the exploration phase quickly, I will try to direct them to looking for an equation for the distance between two points.

SUMMARIZE/SHARE :

When the class comes back together for discussion, we will go over the answers and ask the students to describe their strategy for calculating the fastest routes. Did anybody use a different strategy?

We will then derive the distance formula together using the two points (X,Y) and (A,B).
Leg 1 = X – A
Leg 2 = Y – B
The hypotenuse is $\sqrt{((\text{Leg }1)^2 + (\text{Leg }2)^2)}$
This can be generalized as:
The distance formula follows:
x and y are the x,y coordinates for the first point
a and b are the x,y coordinates for the second point
d is the distance between the points:

$$d = \sqrt{(x - a)^2 + (y - b)^2}$$

Demo Applet:
http://www.learner.org/courses/learningmath/geometry/session6/part_c/distance.html
Student Resource:
http://www.purplemath.com/modules/distform.htm

	<u>Student Notebook Summary:</u> Students will take notes on the derivation of the distance formula and record the formula in their notes. <u>Closure:</u> Pythagoras would be proud of you all. You will have completed another Pythagorean challenge on your way to be inducted into the secret math society of the Pythagoreans. Understanding the distance formula and the right triangle geometries in the coordinate grid will be useful to you in your future.
Assessment	The students will be informally assessed throughout the class with assessment coming from the questions to be asked during the exploration phase listed above. The student lab sheet will be turned in as a class work assessment. Students will hand in homework which is distance formula practice.
Reflections *This section to be completed only if lesson plan is implemented.*	*Student Work Sample 1 – Approaching Proficiency:* *Student Work Sample 2 – Proficient:* *Student Work Sample 3 – Exceeds Proficiency:* *Lesson Implementation:*

Worksheet Coordinate Geometry, Distance Formula

Name:_________________
Date:_______________

Use the coordinate grid below to calculate the distance between the following points using the Pythagorean Theorem. Show your work.

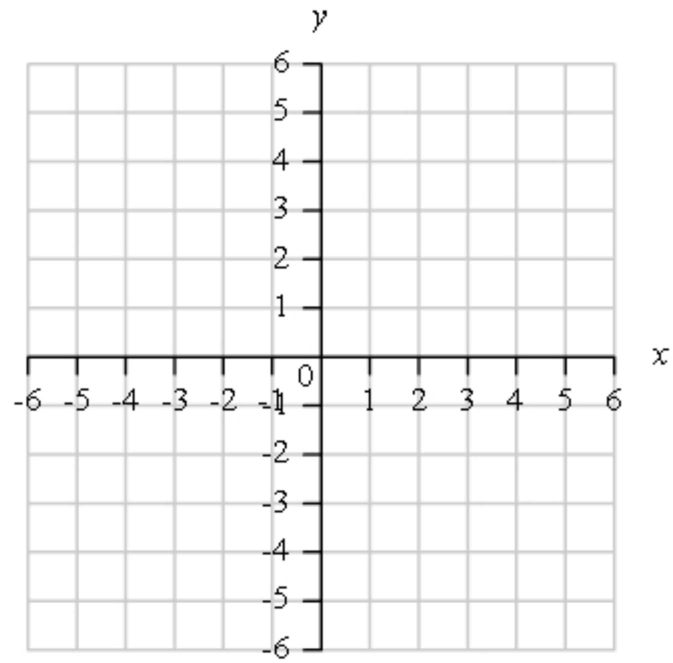
1. (0,0) and (1,1) d = ______________.

2. (2,3) and (-1,-1) d = ______________.

3. (-3,-2) and (5,4) d = ______________.

4. (-3, 4) and (1,-1) d = ______________.

5. (-2, -3) and (-3,5) d = ______________.



Calculate the following using the distance formula.
$d = \sqrt{((x_1 - x_2)^2 + (y_1 - y_2)^2)}$
Show your work:

1. (12,15) and (4,5) d = ______________.

2. (2,3) and (-2,-3) d = ______________.

3. (-5,10) and (5, -10) d = ______________.

4. (-3, 4) and (-6,-8) d = ______________.

5. (36, 40) and (6,5) d = ______________.

Record Sheet for Taxicab Geometry Exploration Phase:

Name:________________________ Date:___________________

Use the Pythagorean Theorem to calculate the fastest route (the diagonal). Draw the right triangles to help you calculate the distances.

Point Leaving From	Point Going to	Distance traveled horizontally	Distance traveled vertically	Diagonal distance calculated with Pythagorean Theorem
B - Bank	C - Starbucks	5	3	$\sqrt{(25 + 9)} = \sqrt{34} = 5.83$
M - Mall	A – Art Museum	1	3	$\sqrt{(1 + 9)} = \sqrt{10} = 3.16$
A – Art Museum	P - Pizza	4	1	$\sqrt{(16 + 1)} = \sqrt{17} = 4.12$
P - Pizza	M - Mall			
M - Mall	R - River			
R – River	H - Hotel			

Lesson 6 - Special Right Triangles, Similarity, Proportions and Scale

<table>
<tr><td>Grade/Content Area</td><td>For ninth grade math students in a pre-algebra (math foundations) class.</td></tr>
<tr><td>Lesson Title</td><td>Pythagorean Challenge 6 - Special Right Triangles
Similarity, Proportions and Scale</td></tr>
<tr><td>State Standards:
GLEs/GSEs
National Content Standards:</td><td>RI Standards:

M(G&M)–8–5 Applies concepts of similarity to determine the impact of scaling on the volume or surface area of three-dimensional figures when linear dimensions are multiplied by a constant factor; to determine the length of sides of similar triangles, or to solve problems involving growth and rate. (Local)

(N&O)–10–4 Accurately solves problems that involve but are not limited to proportional relationships, percents, ratios, and rates. (The problems might be drawn from contexts outside of and within mathematics including those that cut across content strands or disciplines.) (State)

M(G&M)–10–5 Applies concepts of similarity by solving problems within mathematics or across disciplines or contexts. (State)

M(CCR)–8–1 Students will communicate their understanding of mathematics and be able to:
Articulate ideas clearly and logically in both written and oral form.
Present, share, explain, and justify thinking with others and build upon the ideas of others to solve problems.
Use mathematical symbols and notation.
Formulate questions, conjectures, definitions, and generalizations about data, information, and problem situations.
NCTM Communication Standard
communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
analyze and evaluate the mathematical thinking and strategies of others;</td></tr>
<tr><td>Context of the Lesson
Where does this lesson fit in the curriculum and instructional context? Is it the opening of a unit or a series of lessons?</td><td>This lesson extends the concepts of the right triangles to pointing out further special properties of some special right triangles. These special properties will be emphasized. The concept of similarity will be reviewed and connected to useful right triangle proportions. This lesson will be mostly conducted through student centered discussion during the launch phase of the lesson. The exploration is more applied examples to convince the students that the proportions are equal for similar triangles.</td></tr>
</table>

	Prior Knowledge: Students should be familiar with similarity, but it will be reviewed. The technology piece: an online video for reviewing similarity. It is planned for an 82 minute block.
Opportunities to Learn *Differentiation: Materials, Learners and Environments*	**Plans to differentiate instruction:** Students who are struggling with the exploration will be supported in a scaffolded approach. Visual: the video and the applet Kinesthetic: Verbal: partner work will make the experience more enjoyable. Describing their observations and justifying their conclusions is verbal **Accommodations and modifications:** Students with disabilities will work in groups. Peer partners can help with manipulating the square pieces. Student X must be seated in the front of the class during discussions due to vision issues. **Environment factors:** Classroom space needed to allow students to work in groups of 2; desks will be moved together to allow groups to work together. White board or black board space. Computer and projector to display technology applet **Materials:** Exploration for in class activity Notebooks Pencil Computer/laptop Overhead Projector Internet connection to access to: 3) http://www.brainpop.com Similar Figures 4) http://www.mathopenref.com/congruenttriangles.html 5) http://www.mathopenref.com/similartriangles.html
Objectives	Students will: • Understand special right triangles 30-60-90 and 45-45-90 • Develop an understanding of similarity • Review congruence • Apply similarity to find lengths in a similar triangle • Apply proportion to solve problems involving scaled right

	triangles • Explain/communicate the process used during the exploration process
Instructional Procedures	**LAUNCH:** <u>Setting the Context and Activating Prior Knowledge:</u> We will begin student centered discussion. I will place different right triangles on the overhead and project the shadows on the board. There will be a 45-45-90, a 30-60-90 and two other right triangles with different angle measurements. We will discuss what they triangles have in common. Then we will look at the special characteristics of the 45-45-90 and the 30-60-90 triangles. Through analysis of the projections, discussion should lead to: 45-45-90 properties: 1) two legs or sides are equal 2) if doubled, it creates a square 3) the hypotenuse length is equal to $s*\sqrt{2}$ 4) the Pythagorean theorem holds 30-60-90 properties: 1) all sides are different lengths, so it is a scalene triangle 2) if doubled it creates a rectangle 3) the hypotenuse length is equal to $c= 2a$ 4) the Pythagorean theorem holds Other right triangles: 1) all sides are different lengths 2) if doubled it creates a rectangle 3) there is no special relationship of the hypotenuse to the sides 4) the Pythagorean theorem holds Once this is established, we will discuss similarity with the opening question, "Are these right triangles similar?" Discussion will ensue as to whether they are geometrically similar. The points that need to be made is that they are all in the same classification as triangles and even more specifically right triangles, but they are not similar. In order to be similar they have to have the same shape. Do these triangles have the same shape? I will ask the students that say yes, to prove to me that they have the same shape. Ultimately we will conclude that in order for triangles to have the same shape they have to have the same interior angle measurements and proportional sides. Students will be reminded of how to set up a proportion: AB/A'B' = BC/B'C' = CA/C'A' This will be demonstrated by measuring the triangle on the overhead projector and measuring the projected shape and establishing that the proportions are equal and we will determine the scale factor. Students will be asked to measure the projections to keep the class involved. Congruence will be reviewed or introduced here as well. Congruence is the same shape and the same size. Similarity is the same shape different sizes. This will lead to the definition of similar triangles: Definition: Triangles are *similar* if they have the same shape, but

	can be different sizes. (They are still similar even if one is rotated, or one is a mirror image of the other). Similar triangles demonstration applet: http://www.mathopenref.com/similartriangles.html Definition: We examine two triangles which are congruent because all corresponding angles and sides have the same measures. Congruent triangles démonstration applet: http://www.mathopenref.com/congruenttriangles.html If we establish similarity, we can use proportions to determine unknown lengths. The Egyptians knew this and they applied this technique liberally in order to determine a whether something was perpendicular and to measure objects that were too tall to measure by hand. It was applied to early astronomical calculations as well. <u>Establishing the mathematical language:</u> I will be listening and encouraging the following key vocabulary: Interior angles, the sum of the interior angles of a triangle is 90 degrees, geometrically similar, congruent, similar parts, proportion, scale, Auxiliary vocabulary: dilate, compress <u>Issuing the Challenge and setting expectations:</u> In order to be inducted into the Pythagorean Society, you must work alone to establish that you understand how to use proportions to determine lengths of similar right triangles. You will be given a lab sheet with a number of similar triangles and you will be asked to determine the lengths of two of the sides of one of the triangles. This will be in class work, but if it is not finished, it will be homework. **EXPLORATION:** <u>Monitoring/ Observing and Promoting On Task Behavior</u> The students will be given 20 minutes for the exploration. The lab sheet is attached. <u>Asking Probing Questions</u> If the proportion is true for the similar sides of two triangles. Does the same proportion hold for the other sides? Because of the nature of this lab is to convince the students that proportionality holds, more scaffolded questions will be asked to help any students who need it. <u>Issuing Extra Challenges</u> For students who finish quickly (for differentiation), they will be asked to identify any similar triangles (or polygons) in the classroom and justify those that they find. Since this is homework, they can use this challenge to occupy their time and finish their

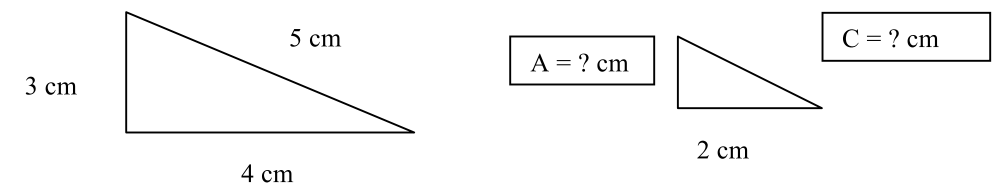
	homework early. **SUMMARIZE/SHARE EXPLORATION :** The class will be called back together to go over the first few problems on the sheet to see that the students understand the proportions. This should be entered in their notebooks either during the launch phase because they students will need it for the exploration and for homework. <u>Student Notebook Summary:</u> 45-45-90 properties: 1) two legs or sides are equal 2) if doubled, it creates a square 3) the hypotenuse length is equal to s*√2 4) the Pythagorean theorem holds 30-60-90 properties: 1) all sides are different lengths, so it is a scalene triangle 2) if doubled it creates a rectangle 3) the hypotenuse length is equal to c= 2a 4) the Pythagorean theorem holds Other right triangles: 1) all sides are different lengths 2) if doubled it creates a rectangle 3) there is no special relationship of the hypotenuse to the sides 4) the Pythagorean theorem holds Example of similar triangles Picture and congruent angles identified AB/A'B' = BC/B'C' = CA/C'A' = a number p $pa^2 + pb^2 = pc^2$ dilation if p >1 and compression if p<1 Example of congruent triangles Picture and congruent angles identified AB/A'B' = BC/B'C' = CA/C'A' = 1 <u>Closure:</u> Pythagoras would be proud of you all. You will have completed another Pythagorean challenge on your way to be inducted into the secret math society of the Pythagoreans. Understanding the concept of similar triangles and the right triangle geometries will be useful to you in your future.
Assessment	The students will be informally assessed throughout the class with

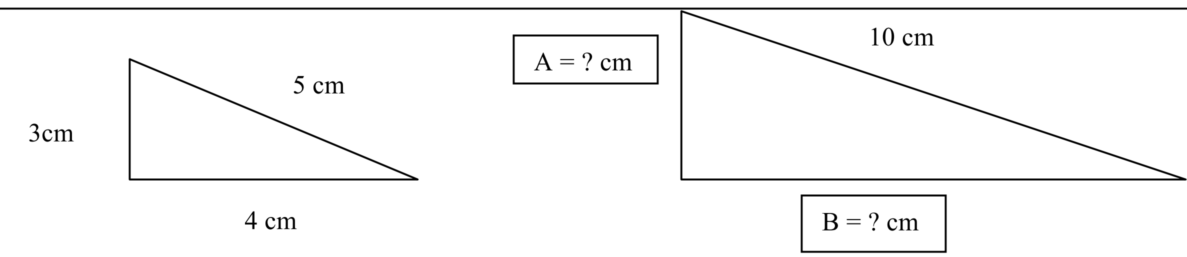
	assessment coming from the questions to be asked during the exploration phase listed above. The student lab sheet will be turned in as a class work assessment. Try to identify similar triangles or similar polygons at home, talk to people you know to help you identify similar shapes. List three and bring to class the following day.
Reflections *This section to be completed only if lesson plan is implemented.*	*Student Work Sample 1 – Approaching Proficiency:* *Student Work Sample 2 – Proficient:* *Student Work Sample 3 – Exceeds Proficiency:* *Lesson Implementation:*

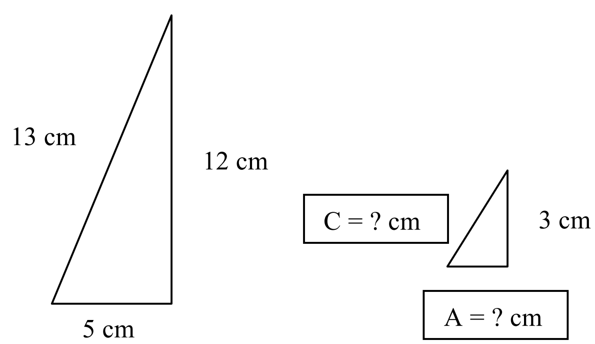
Similar Right Triangles Lab Sheet

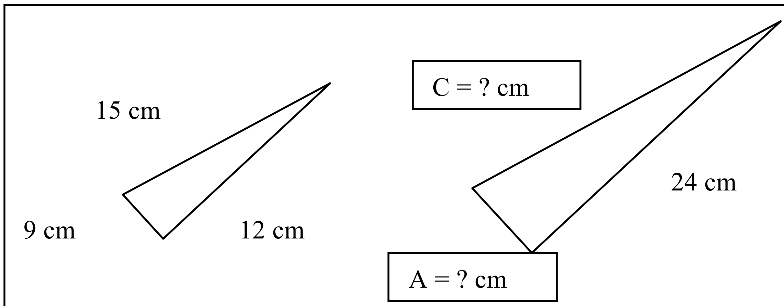
Name:________________________ Date:___________________

The triangle pairs below are similar. Since they are similar determine the unknown lengths by using proportions.









Lesson 7 – Measuring the Height of a Grecian Temple

Unit Plan - Lesson 7

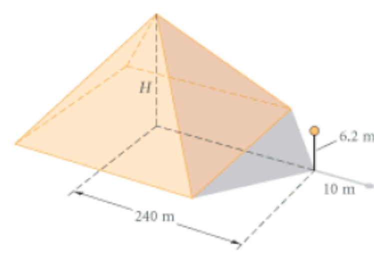
Grade/Content Area	For ninth grade math students in a pre-algebra (math foundations) class.
Lesson Title	*Measuring the height of an Grecian Temple*
State Standards: GLEs/GSEs **National Content Standards**:	RI Standards: M(G&M)–8–5 Applies concepts of similarity to determine the impact of scaling on the volume or surface area of three-dimensional figures when linear dimensions are multiplied by a constant factor; to determine the length of sides of similar triangles, or to solve problems involving growth and rate. (Local) (N&O)–10–4 Accurately solves problems that involve but are not limited to proportional relationships, percents, ratios, and rates. (The problems might be drawn from contexts outside of and within mathematics including those that cut across content strands or disciplines.) (State) M(G&M)–10–5 Applies concepts of similarity by solving problems within mathematics or across disciplines or contexts. (State) M(CCR)–8–1 **Students will communicate their understanding of mathematics** and be able to: • Articulate ideas clearly and logically in both written and oral form. • Present, share, explain, and justify thinking with others and build upon the ideas of others to solve problems. • Use mathematical symbols and notation. • Formulate questions, conjectures, definitions, and generalizations about data, information, and problem situations. NCTM Communication Standard • communicate their mathematical thinking coherently and clearly to peers, teachers, and others; • analyze and evaluate the mathematical thinking and strategies of others;
Context of the Lesson *Where does this lesson fit in the curriculum and instructional context? Is it the opening of a unit or a series of lessons?*	This lesson continues the use of proportions of similar right triangles for practical applications. We will hope for a sunny day in order to measure a tree or something outside. If there is not enough sun for the shadow project we will conduct it inside with flashlights. There is also a way to perform this experiment with a mirror. Prior Knowledge: Students should be familiar with similarity, calculating

	proportions and solving for an unknown length. The technology piece: an online video for reviewing similarity. It is planned for an 82 minute block.
Opportunities to Learn *Differentiation: Materials, Learners and Environments*	**Plans to differentiate instruction:** Students who are struggling with the exploration will be supported in a scaffolded approach. Visual: the video and the applets Kinesthetic: walking outside and measuring Verbal: partner work will make the experience more enjoyable. Describing their observations and justifying their conclusions is verbal **Accommodations and modifications:** Students with disabilities will work in groups. Peer partners can help with manipulating the square pieces. Student X must be seated in the front of the class during discussions due to vision issues. **Environment factors:** Classroom space needed to allow students to work in groups of 2; desks will be moved together to allow groups to work together. White board or black board space. Computer and projector to display technology applet **Materials:** Exploration for in class activity Notebooks Pencil Computer/laptop
Objectives	Students will: • Set up and solve proportions • Application of similar triangle proportions for indirect measurement • Explain/communicate the process used during the exploration process
Instructional Procedures	**LAUNCH:** Setting the Context and Activating Prior Knowledge: The students were asked to find example of similar triangles or polygons at home and bring in three examples. We will review the concept of similar triangles and go over any examples the students brought in from home. If we establish similarity, we can use proportions to determine

unknown lengths. The Egyptians knew this and they applied this technique liberally in order to determine a whether something was perpendicular and to measure objects that were too tall to measure by hand. It was applied to early astronomical calculations as well.

Historically mathematicians were hired to solve problems. A man named Thales of Miletos (625 – 547 B.C.) used the concept of similar right triangles to measure the heights of Egyptian pyramids. Thales observed that a man's shadow was the same length as himself at a certain time of the day. He reasoned that the shadow of the Great Pyramid would be the same as the height of the pyramid.

While vacationing in Egypt, the Greek mathematician Thales calculated the height of the Great Pyramid. According to legend, Thales placed a pole at the tip of the pyramid's shadow and used similar triangles to calculate its height. This involved some estimating because he was unable to measure the distance from directly beneath the height of the pyramid to the tip of the shadow. From the diagram, explain his method. Calculate the height of the pyramid from the information given in the diagram.



Retrieved 11/28/09 from http://www.redmond.k12.or.us/14552011718214563/lib/14552011718214563/Lesson_11.3.pdf

Establishing the mathematical language:
I will be listening and encouraging the following key vocabulary:
Proportion, indirect measurement, similar right triangles.
Auxiliary vocabulary:

Issuing the Challenge and setting expectations:
You will work in groups of two or three. Your Pythagorean challenge today is to use the concept of similar triangles to measure a flagpole or tree outside using the proportions of shadows. This concept works because similar right triangles are formed based on the angle of the sun. Record your observations and measurements on the lab sheet provided.

EXPLORATION:
Monitoring/ Observing and Promoting On Task Behavior
How can you get the proportion you need for the indirect measurement?

Asking Probing Questions
When do we need to use the concept of similarity and indirect measurement? Why is it helpful? [When we don't know the height

of a tall object, for example, we can use shadows and indirect measurement to help us determine the height.]

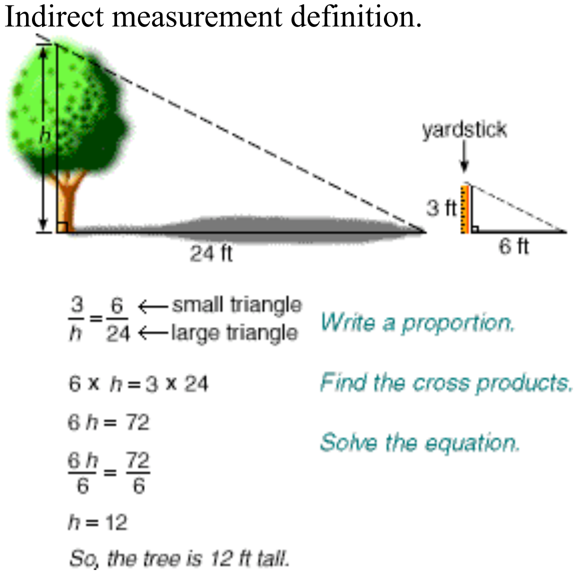
Why are we able to use shadows to make these predictions about heights? [Assuming we make several measurements at the same time, the lengths of the shadows should be proportional, thereby allowing us to make predictions about heights of tall objects.]

Why are similar figures proportional? [The ratios of the length and height of one object is equal to the ratio of the length and height of another object, when they are similar.]

<u>Issuing Extra Challenges</u>
For students who finish quickly (for differentiation), they will be asked to help other student groups.

SUMMARIZE/SHARE EXPLORATION :
The class will be called back together to go over their observations and identify examples that the students measured indirectly.
Indirect measurement: The technique of using similar figures and proportions to find a measure.

<u>Student Notebook Summary:</u>
Indirect measurement definition.



http://www.encyclo.co.uk/define/indirect%20measurement

Picture and congruent angles identified
AB/A'B' = BC/B'C' = CA/C'A' = a proportion

<u>Closure:</u>
Pythagoras would be proud of you all. You will have completed

	another Pythagorean challenge on your way to be inducted into the secret math society of the Pythagoreans. This method of indirect measurement will earn you money in the future. Many jobs use similar methods to obtain measurements like surveyors.
Assessment	The students will be informally assessed throughout the class with assessment coming from the questions to be asked during the exploration phase listed above. The student lab sheet will be turned in as a class work assessment.
Reflections *This section to be completed only if lesson plan is implemented.*	*Student Work Sample 1 – Approaching Proficiency:* *Student Work Sample 2 – Proficient:* *Student Work Sample 3 – Exceeds Proficiency:* *Lesson Implementation:*

Measuring a Grecian Temple Lab Sheet

Name: _________________ Date:______________

Measure your height: _____________ Set up the proportion:
Measure your shadow: ____________

Measure the shadow of a smaller object that you can measure the shadow: _______________

How tall is the object using the proportion above: ____________________________________

Now measure the height of the object: ____

How does your indirect measurement relate to the actual height of the object?
If there is any difference, talk with your partner about what could have caused error.

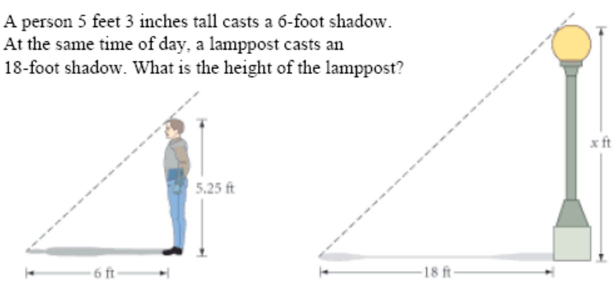
Measure the shadow of a tree or a flagpole:_________________________

Apply the proportion:____

What is the height of the tree or the flagpole (Grecian Temple) ? _______

EXAMPLE

A person 5 feet 3 inches tall casts a 6-foot shadow. At the same time of day, a lamppost casts an 18-foot shadow. What is the height of the lamppost?



Solution

The light rays that create the shadows hit the ground at congruent angles. Assuming both the person and the lamppost are perpendicular to the ground, you have similar triangles by the AA Similarity Conjecture. Solve a proportion that relates corresponding lengths.

You could also choose to set up a proportion using ratios of side lengths *between* similar triangles.

$\frac{5.25}{6} = \frac{x}{18}$ Ratios of side lengths *within* similar triangles are equal.

$18 \cdot \frac{5.25}{6} = x$ Multiply both sides by 18.

$15.75 = x$ Simplify left side.

The height of the lamppost is 15 feet 9 inches.

Lesson 8 – Utilizing the Pythagorean Secrets

Unit Plan - Lesson 8

Grade/Content Area	For ninth grade math students in a pre-algebra (math foundations) class.
Lesson Title	*Pythagorean Challenge 8* *Utilizing the Pythagorean Secrets*
State Standards: GLEs/GSEs **National Content Standards**:	RI Standards: M(G&M)–8–5 Applies concepts of similarity to determine the impact of scaling on the volume or surface area of three-dimensional figures when linear dimensions are multiplied by a constant factor; to determine the length of sides of similar triangles, or to solve problems involving growth and rate. (Local) (N&O)–10–4 Accurately solves problems that involve but are not limited to proportional relationships, percents, ratios, and rates. (The problems might be drawn from contexts outside of and within mathematics including those that cut across content strands or disciplines.) (State) M(G&M)–10–6 Solves problems involving perimeter, circumference, or area of two-dimensional figures (including composite figures) or surface area or volume of three-dimensional figures (including composite figures) within mathematics or across disciplines or contexts. (State) M(CCR)–8–1 **Students will communicate their understanding of mathematics** and be able to: • Articulate ideas clearly and logically in both written and oral form. • Present, share, explain, and justify thinking with others and build upon the ideas of others to solve problems. • Use mathematical symbols and notation. • Formulate questions, conjectures, definitions, and generalizations about data, information, and problem situations. NCTM Communication Standard • communicate their mathematical thinking coherently and clearly to peers, teachers, and others; • analyze and evaluate the mathematical thinking and strategies of others;
Context of the Lesson *Where does this lesson fit in the curriculum and instructional context? Is it the*	This lesson continues the use of proportions of similar right triangles for practical applications. Shapes will be broken down into squares, rectangles, and right triangles in order to determine area and perimeter. Many word problems will be utilized. Before we begin working through a number of different problems, we will create a study sheet to help us remember all the useful properties of right triangles and the formulas we have

opening of a unit or a series of lessons?	derived from the very useful Pythagorean Theorem. Students will use exploration time to consolidate all that they have learned in a study guide. We will then work on application, extension and connection problems from the Connected Mathematics Looking for Pythagoras unit. We will solve problems together on the overhead and we will solve problems in small cooperative groups. This will enable the students to connect all that was learned in this unit and prepare them for the summative assessment. Prior Knowledge: Students should be familiar with similarity, calculating proportions, solving for an unknown length, and a familiarity with perimeter and area. It is planned for an 82 minute block.
Opportunities to Learn *Differentiation: Materials, Learners and Environments*	**Plans to differentiate instruction:** Students who are struggling with the exploration will be supported in a scaffolded approach. Small cooperative groupings may be used. **Accommodations and modifications:** Students with disabilities will work in groups. Peer partners can help with manipulating the square pieces. Student X must be seated in the front of the class during discussions due to vision issues. **Environment factors:** Classroom space needed to allow students to work in groups of 2; desks will be moved together to allow groups to work together. White board or black board space. Computer and projector to display technology applet **Materials:** Exploration for in class activity Notebooks Pencil Computer/laptop Overhead Projector for problem solutions
Objectives	Students will: • Analyze triangles and other shapes with right triangles • Apply the Pythagorean Theorem to find missing sides to determine perimeter or area of two dimensional shapes • Explain/communicate the process used during the

	exploration process • Create a study guide for use on the exercises and the summative assessment
Instructional Procedures	**LAUNCH:** <u>Setting the Context and Activating Prior Knowledge:</u> The students will be informed that they are embarking on the next Pythagorean challenge. They have almost completed all the necessary challenges to become part of the secret society, Pythagoreans. We must organize all we have learned to date. Students will be given time to create a study guide. The study guide should include the secrets we have learned to date. <u>Establishing the mathematical language:</u> None at this time <u>Issuing the Challenge and setting expectations:</u> You will work alone in creating the study guide. The study guide should include your tool box for analyzing triangles and other shapes. The formulas you learned will help you in the problems we will work on during this lesson and the next. **EXPLORATION:** <u>Monitoring/ Observing and Promoting On Task Behavior</u> Students will be monitored for on-task behavior and encouraged to put all the important information on the study guide. <u>Asking Probing Questions</u> What are the important tools you have learned, that will help you determine areas and perimeters of odd shapes? What formulas are important? <u>Issuing Extra Challenges</u> For students who finish quickly (for differentiation), they will be asked to help other student groups. **SUMMARIZE/SHARE EXPLORATION :** The class will be called back together to go over what tools they felt were important to include on their study guide. We will begin going over analyzing triangles. Equilateral triangle: Split it into two right triangles; use deductive reasoning to get two sides of the right triangles which allows you to find the leg of the right triangle which is the height of the triangle. Then the area can be obtained by ½ b h.

	The remainder of class and the following day, we will work on application, extension and connection problems from the Connected Mathematics Looking for Pythagoras unit. We will solve problems together on the overhead and we will solve problems in small cooperative groups. This will enable the students to connect all that was learned in this unit and prepare them for the summative assessment. <u>Student Notebook Summary:</u> The study guide will be stored in the notebook. <u>Closure:</u> We will continue to apply the properties of right triangles and the Pythagorean Theorem to a number of applications and problems tomorrow.
Assessment	The students will be informally assessed throughout the class with assessment coming from the questions to be asked during the exploration phase listed above. The study guide will be assessed to ensure the students are ready for the summative assessment. The students will have homework, see attached.
Reflections *This section to be completed only if lesson plan is implemented.*	*Student Work Sample 1 – Approaching Proficiency:* *Student Work Sample 2 – Proficient:* *Student Work Sample 3 – Exceeds Proficiency:* *Lesson Implementation:*

Solve the problem. If necessary, round your answers to 1 decimal place.

1) On a sunny day, a flag pole and its shadow form the sides of a right triangle. If the hypotenuse is 50 m long and the shadow is 40 m, how tall is the flag pole?

2) On a sunny day, a tree and its shadow form the sides of a right triangle. If the hypotenuse is 52 m long and the tree is 48 m tall, how long is the shadow?

3) A car dealer advertised a big sale by stretching a string of banners from the top of the building to the edge of the driveway. If the building is 26 m high and the driveway is 49 m from the building, how long is the string of banners?

4) A building has a ramp to its front doors to accommodate the handicapped. If the distance from the building to the end of the ramp is 16 feet and the height from the ground to the front doors is 7 feet, how long is the ramp?

5) An airport has a sloping ramp from the terminal down to the door of an airplane. The door of the airplane is 44 feet away from the terminal (at the point where the ramp starts) and is 6 feet below the terminal side of the ramp. How long is the ramp?

Worksheet from Lesson Planet website retrieved 11/28/09
http://itech.pjc.edu/gbloxom/mat1033/handouts/ConvertedFiles/mat10337.7.pdf

Lesson 9 – The Ultimate Pythagorean Secret

Unit Plan - Lesson 9

Grade/Content Area	For ninth grade math students in a pre-algebra (math foundations) class.
Lesson Title	*Pythagorean Challenge 9* *The Ultimate Pythagorean Secret*
State Standards: GLEs/GSEs **National Content Standards**:	RI Standards: M(N&O)–8–2 Demonstrates understanding of the relative magnitude of numbers by ordering or comparing rational numbers, common irrational numbers (e.g., ,), numbers with whole number or fractional bases and whole number exponents, square roots, absolute values, integers, or numbers represented in scientific notation using number lines or equality and inequality symbols. (Local) NCTM Communication Standard • communicate their mathematical thinking coherently and clearly to peers, teachers, and others; • analyze and evaluate the mathematical thinking and strategies of others;
Context of the Lesson *Where does this lesson fit in the curriculum and instructional context? Is it the opening of a unit or a series of lessons?*	This lesson addresses irrational numbers as they are encountered when working with square roots. The historical story will be told in order to weave some history and storytelling into the lesson. Prior Knowledge: Students should be familiar with the real numbers, integers, and rational numbers. It is planned for an 82 minute block.
Opportunities to Learn *Differentiation: Materials, Learners and Environments*	**Plans to differentiate instruction:** Students who are struggling with the exploration will be supported in a scaffolded approach. Other students will be challenged if necessary. **Accommodations and modifications:** Students with disabilities will work in groups. Peer partners can help with manipulating the square pieces. Student X must be seated in the front of the class during discussions due to vision issues.

<table>
<tr><td></td><td>Environment factors:
Classroom space needed to allow students to work in groups of 2; desks will be moved together to allow groups to work together.
White board or black board space.
Computer and projector

Materials:
Exploration for in class activity
Notebooks
Pencil
Calculators
Computer/laptop
Overhead Projector for example demonstrations
Internet access:
http://www.brainpop.com/math/numbersandoperations/rationalandirrationalnumbers/preview.weml</td></tr>
<tr><td>Objectives</td><td>Students will:
Identify irrational numbers
Understand relationship to rational numbers and the real number system
Locate irrational numbers on a number line</td></tr>
<tr><td>Instructional Procedures</td><td>LAUNCH:
<u>Setting the Context and Activating Prior Knowledge:</u>
The students will be informed that they are embarking on the next Pythagorean challenge. They have almost completed all the necessary challenges to become part of the secret society of the Pythagoreans.
The last secret will be revealed during this lesson.

A historical account of the Pythagorean Society will be given regarding their viewpoint that all numbers can be represented in a rational form. When they discovered that there were some numbers that could not be represented as rational numbers, they called those numbers irrational numbers and swore not to reveal their knowledge of irrational numbers.

The square root of 2 is in fact not a rational number, as was discovered by the Pythagorean Brotherhood. According to one legend, this idea was so shocking to Pythagoras that he put the discoverer to death. Another version says that Pythagoras declared the irrationality of the square root of 2 to be a secret, not to be revealed to anyone outside the Brotherhood. When one of the members dared to defy Pythagoras' decree and inform the outside world of the discovery, he was killed.

Discussion will occur to assess student awareness of rational and</td></tr>
</table>

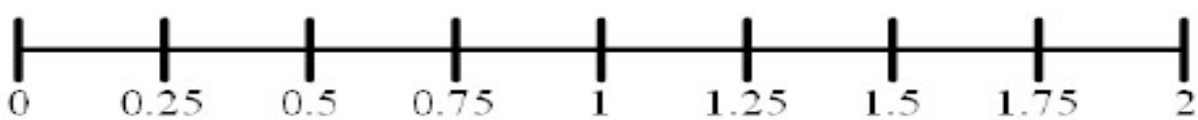
	irrational numbers. <u>Establishing the mathematical language:</u> Rational numbers, real numbers, irrational numbers <u>Issuing the Challenge and setting expectations:</u> You will work with a partner. You will be given a lab sheet. You will explore the difference between irrational and rational numbers. We will come together as a class and discuss what you have discovered. What else makes an irrational number irrational? Write the numbers from your calculator to the 1/100,000th place and place the numbers on the number line provided on the lab worksheet. **EXPLORATION:** <u>Monitoring/ Observing and Promoting On Task Behavior</u> Students will be monitored for on-task behavior and encouraged with support. <u>Asking Probing Questions</u> <u>Issuing Extra Challenges</u> For students who finish quickly (for differentiation), they will be asked to help other student groups. Also students will be asked to estimate the square roots of some of the numbers considered to be irrational. **SUMMARIZE/SHARE EXPLORATION :** When the class comes together we will plot the values of the irrational and rational numbers from the exploration on the number line. Through student centered discussion we will begin to discuss observations as a class to try to narrow down the differences between the decimal representation of irrational numbers and rational numbers. Rational decimal representations are those that end or terminate. Some rational number decimal representations repeat forever. Ex. 1/3, 2/3 Irrational number decimal representations do not repeat or terminate. Ex. $\sqrt{2}$ $\sqrt{3}$ π BrainPop video to summarize irrational numbers http://www.brainpop.com/math/numbersandoperations/rationalandirrationalnumbers/preview.weml <u>Student Notebook Summary:</u> *Rational numbers*: (a ratio) Any number that can be made by dividing one integer by another.

	The word comes from "ratio" where b cannot be = 0. Examples: 1/2 is a rational number (1 divided by 2, or the ratio of 1 to 2) 0.75 is a rational number (3/4) 1 is a rational number (1/1) 2 is a rational number (2/1) 2.12 is a rational number (212/100) -6.6 is a rational number (-66/10) *Irrational numbers*: (not a ratio) An Irrational Number is a number that cannot be written as a simple fraction - the decimal goes on forever without repeating. Example: π (Pi) is an irrational number. The value of Pi is 3.1415926535897932384626433832795 (and more...) There is no pattern to the decimals, and you cannot write down a simple fraction that equals Pi. The popular approximation of 22/7 = 3.1428571428571... Is close but not accurate. Closure: We will continue to apply the properties of right triangles and the Pythagorean Theorem to a number of applications and problems tomorrow.
Assessment	The students will be informally assessed throughout the class with assessment coming from the questions to be asked during the exploration phase listed above. The students will have homework, see attached.
Reflections *This section to be completed only if lesson plan is implemented.*	*Student Work Sample 1 – Approaching Proficiency:* *Student Work Sample 2 – Proficient:* *Student Work Sample 3 – Exceeds Proficiency:* *Lesson Implementation:*

Exploration Irrational and Rational Numbers

Name:___________________________ Date:______________________

Real Number	Decimal Representation	Terminating? Repeating? Non-repeating?	Observations? Rational? Irrational? Why?
2/3			
1/2			
3/4			
1/9			
2/9			
3/9			
1/11			
√2			
π			
√5			
√16			
2			
5/6			
7/7			



Formative Assessment

The formative assessments are indicated within each lesson plan. Three quizzes are placed throughout the unit in order to assess student's grasp of the important topics.

Quiz 1

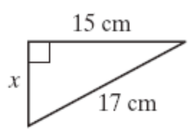
Can you form a right triangle with the three lengths given? Show your work.

1. 20, 21, 29 __________ **2.** 7, 11, 12 __________ **3.** $10, 2\sqrt{11}, 12$ __________

4. 28, 45, 53 __________ **5.** $9, \sqrt{10}, 10$ __________ **6.** 10, 15, 20 __________

Find each missing length to the nearest tenth of a unit.

7.



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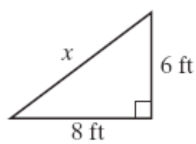
8.



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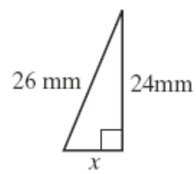
9.



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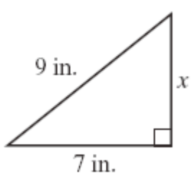
10.



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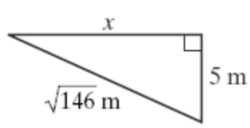
11.



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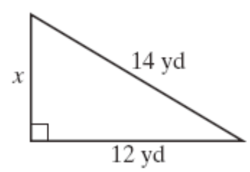
12.



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Use the triangle at the right. Find the missing length to the nearest tenth of a unit.

13. $a = 6$ m, $b = 9$ m

$c \approx$ __________

14. $a = 19$ in., $c = 35$ in.

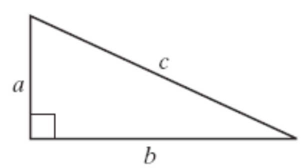
$b \approx$ __________

15. $b = 24$ cm, $c = 32$ cm

$a \approx$ __________

16. $a = 14$ ft, $c = 41$ ft

$b \approx$ __________



17. A rectangular park measures 300 ft by 400 ft. A sidewalk runs diagonally from one corner to the opposite corner. Find the length of the sidewalk.

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Lesson Planet http://teachers.olatheschools.com/fishelprealgebra/files/2009/03/112-practice.pdf

Quiz 2

Kuta Software - Infinite Pre-Algebra

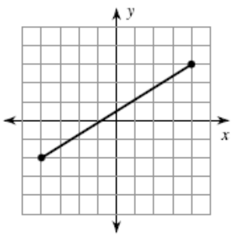
Name________________________________

The Distance Formula

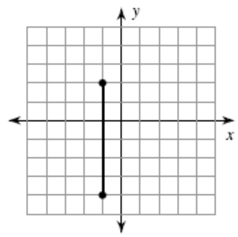
Date______________ Period____

Find the distance between each pair of points.

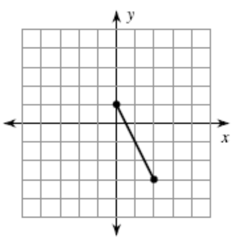
1)



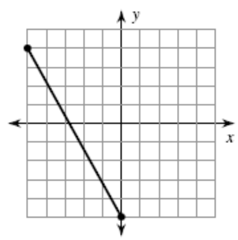
2)



3)



4)



5) (−1, 2), (2, −4)

6) (4, 3), (−3, 4)

7) (0, 4), (2, 3)

8) (4, 0), (−4, 1)

9) (12, 12), (−3, 1)

10) (1, −9), (6, −6)

11) (5, −10), (−5, 4)

12) (5, 5), (−6, −4)

Lesson Planet
http://www.kutasoftware.com/FreeWorksheets/PreAlgWorksheets/Distance%20Formula.pdf

Kuta Software - Infinite Pre-Algebra

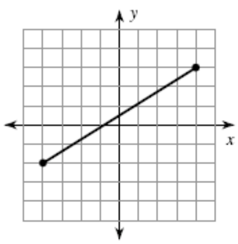
Name________________________________

The Distance Formula

Date_______________ Period___

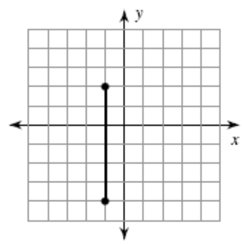
Find the distance between each pair of points.

1)



9.433

2)



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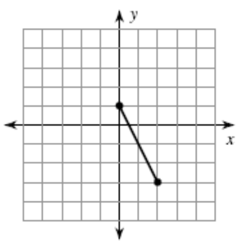
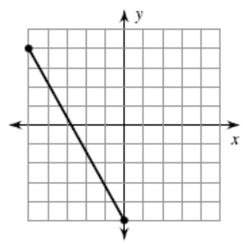
3)



4.472

4)



10.295

5) $(-1, 2)$, $(2, -4)$

6.708

6) $(4, 3)$, $(-3, 4)$

7.071

7) $(0, 4)$, $(2, 3)$

2.236

8) $(4, 0)$, $(-4, 1)$

8.062

9) $(12, 12)$, $(-3, 1)$

18.601

10) $(1, -9)$, $(6, -6)$

5.83

11) $(5, -10)$, $(-5, 4)$

17.204

12) $(5, 5)$, $(-6, -4)$

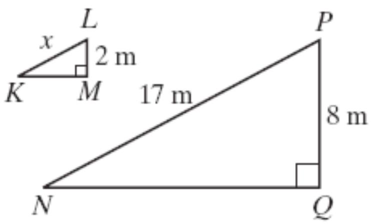
14.212

Quiz 3

Name:__________________ Date:____________

Write a proportion and find the value of each x.

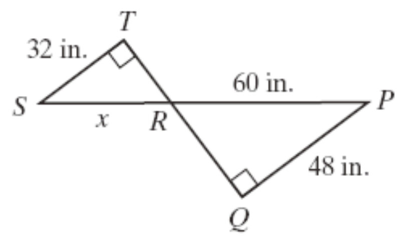
1. $\triangle KLM \sim \triangle NPQ$



Proportion: ____________________

$x =$ ____________________

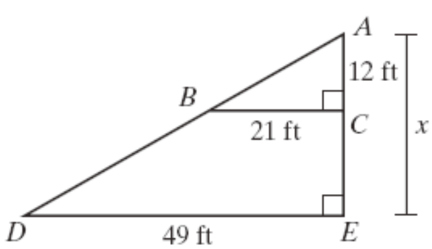
2. $\triangle RST \sim \triangle RPQ$



Proportion: ____________________

$x =$ ____________________

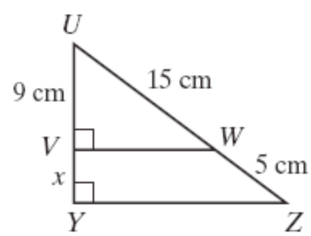
3. $\triangle ABC \sim \triangle ADE$



Proportion: ____________________

$x =$ ____________________

4. $\triangle UVW \sim \triangle UYZ$

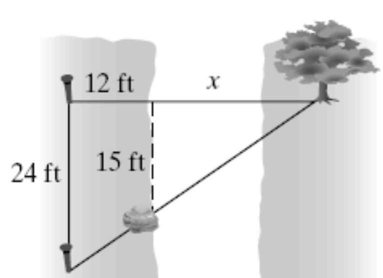


Proportion: ____________________

$x =$ ____________________

Solve. Show the proportion you use.

5. A surveyor needs to find the distance across a canyon. She finds a tree on the edge of the canyon and a large rock on the other edge. The surveyor uses stakes to set up the similar right triangles shown. Find the distance from the tree to the other side of the canyon, x.



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Summative Assessment

The summative assessment is a performance assessment that will be administered in class. A rubric will not be used; rather a grade will be given based on the number of calculations done correctly. Showing work will be imperative.

Summative Assessment – Pythagorean Monument

Name:_____________________________ Date:___________________

Referring to the sketch of a proposed monument to Pythagoras on the previous page, answer the following questions. Show all work.

Calculate the area of the monument. Show all your calculations on the diagram.

Calculate the perimeter of the monument (the yellow lined area): Show all your calculations on the diagram.

From the shapes that form the monument, which shape is a perfect square?

Which shape is a 45-45-90 degree triangle?

Which segment of the perimeter is an irrational number?

Name one segment that has a rational number as a length.

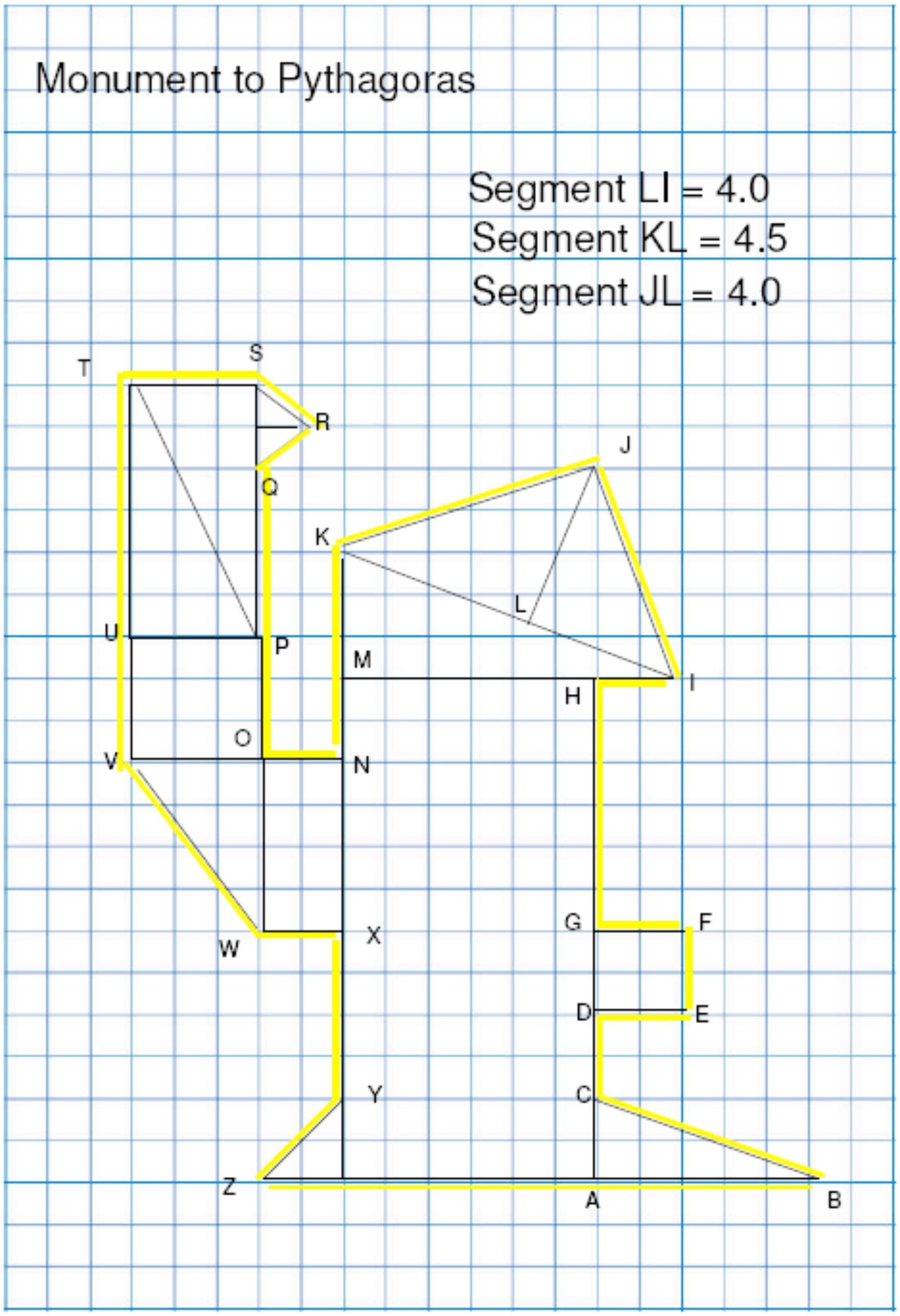
Name two congruent triangles.

Name two similar triangles.

Determine the distance between two points: (-3,2) (6,8)

Determine the distance between two points: (-3,-5) (0,-1)

What would you inscribe on the monument to Pythagoras?



Monument to Pythagoras
Segment LI = 4.0
Segment KL = 4.5
Segment JL = 4.0
T
S
R
J
Q
K
L
U
P
M
H
I
O
V
N
G
F
W
X
D
E
Y
C
Z
A
B

References

Lappan G., et al. (2002) *Connected Mathematics Geometry Looking for Pythagoras*. Prentice Hall. Needham, MA.

Price, J., et al. (1992) *Merrill Pre-Algebra, A Transition to Algebra*. Glencoe Division of Macmilliam/McGraw Hill Publishing Company. Columbus, OH.

www.lessonplanet.com was utilized for some worksheets. They are referenced on the worksheets.

Various online resources were utilized but are referenced within the lesson plans.